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A MARKOV MODEL FOR PARAMETRIC SENSITIVITY ANALYSIS OF *CRUSADER* EFFECTIVENESS

by

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A MARKOV MODEL FOR PARAMETRIC SENSITIVITY ANALYSIS OF *CRUSADER* EFFECTIVENESS

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This thesis presents a Markov model for analyzing the sensitivity of *Crusader* effectiveness to parametric changes in *Crusader* capabilities. *Crusader* activities are represented as finite-state, discrete-time Markov chains. A series of supporting models for some of the Markov model parameters provide for the desired sensitivity analysis on specified engineering characteristics of a *Crusader*. A Microsoft *Excel V7.0* spreadsheet serves as the user interface, computational tool for the models. A Microsoft *Visual Basic* program manipulates the transition probability matrices then returns the results of the Markov model. Available measures of effectiveness include the time until *Crusader*s are killed by enemy counterfire and the number of fire missions executed and lost by *Crusader*s. A demonstration of the spreadsheet implementation shows the models can be used for the desired sensitivity analysis. While creating a model for *Crusader* was the motivation for this thesis, the model can be used to conduct similar analysis on current or competing field artillery systems.

THESIS DISCLAIMER

The reader is cautioned that models and computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the models provide accurate results and the programs are free of computational and logic errors, they must be further validated and verified. The completion of these tasks is left for further research. Any application of these modes and programs without additional validation and verification is at the risk of the user.

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LIST OF ACRONYMS

ADA	Air Defense Artillery
AGM	Attack Guidance Matrix
ALDT	Administrative and Logistic Down-time
APC	Armored Personnel Carrier
ATR	Ammunition Transfer Rate
CBSI	Crusader Battalion Spreadsheet Implementation
COEA	Cost and Operational Effectiveness Analysis
CTM	Crusader Timeline Model
DPICM	Dual Purpose - Improved Conventional Munitions
DT&E	Developmental Tests and Evaluation
FAASV	Field Artillery Ammunition Supply Vehicle
FARV	Future Armored Resupply Vehicle
GPS	Global Positioning System
HE	High Explosive
IDA	Institute for Defense Analysis
LRP	Logistics Resupply Point
MMC	Markov Model for Crusader
MMCA	Markov Model for Crusader Availability
MRL	Multiple Rocket Launcher
MTBEFF	Mean-Time-Between-Essential-Function-Failure
MTTR	Mean-Time-to-Repair
MVR	Maneuver Forces
OP/CP	Observation Post/Command Post
ORD	Operational Requirements Document
OT&E	Operational Tests and Evaluation
OTEC	Operational Test and Evaluation Concept for Crusader
POC	Platoon Operations Center

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EXECUTIVE SUMMARY

The U. S. Army has identified a requirement for a new field artillery weapon system to support maneuver forces in battle in the next century. The *Crusader* and its companion vehicle, the *Future Armored Resupply Vehicle* (FARV) are under development to fulfill the requirement. Results of the initial Developmental Test and Evaluation (DT&E) and Operational Test and Evaluation (OT&E) were provided in the first Cost and Operational Effectiveness Analysis (COEA). The COEA identified *Crusader* mobility, range, rate of fire, and the FARV ammunition transfer rate (ATR), among others, as major drivers of the *Crusader*/FARV combat effectiveness results. However, the COEA did not provide sensitivity analyses to show incremental changes in combat effectiveness for parametric changes in the *Crusader* capabilities. Continued DT&E/OT&E requires the use of transparent, low-resolution analytical modeling to conduct the desired sensitivity analysis on specified engineering characteristics of the *Crusader*/FARV.

The Markov model developed here, called the Markov Model for *Crusader* (MMC), provides for analysis of the productivity and lifetime of a *Crusader* pair; productivity is measured in the number of fire missions executed by *Crusaders*. The model incorporates the arrival of different fire mission types, *Crusader* availability, and death of a *Crusader* pair as a result of enemy counterfire. During its lifetime, a *Crusader* pair will wait for fire missions, execute survivability moves, conduct resupply and administrative activities, and execute fire missions until the pair is killed. The MMC is a low-resolution, analytical representation of the life and productivity of a *Crusader*.

Enhancement of the MMC by the development of a series of supporting models for some of the MMC parameters provide for the desired sensitivity analysis. These models explicitly account for several engineering characteristics of the *Crusader*/FARV: ammunition transfer rate, rate of fire, mobility, others. These supporting models provide the MMC the explicit capability to analyze the leverage a particular engineering characteristic has on *Crusader* effectiveness. Given the values for these characteristics, the calculations using the MMC yield measures of *Crusader* effectiveness in providing field artillery support to U. S. maneuver forces.

Sensitivity to the model parameters may be measured by several results of calculations using the MMC: 1) the expected time until a particular number of *Crusaders* is killed; 2) the expected number of fire missions executed by the *Crusader* pairs; 3) the expected number of fire missions not executed as a result of *Crusaders* pairs being killed by the enemy during firing; 4) the expected number of fire missions executed and lost by *Crusaders* at any finite time *t*; 5) the expected number and variance of the number of *Crusader* pairs surviving at any finite time *t*. The MMC provides for the analysis of a variety of operationally significant results.

The *Crusader* Battalion Spreadsheet Implementation (CBSI) serves as the user interface, computational tool for the models. Microsoft *Excel V7.0* was used for the CBSI. The Microsoft *Visual Basic* programming language was used for particular operations within the CBSI. Provided the input parameters specified for the MMC, the CBSI returns the results of calculations using the MMC. A demonstration of the CBSI shows the Markov model can be used for the desired sensitivity analysis and, when compared to similar results of the COEA, provides "sensible" and "explainable" results with minimal computational effort. Additionally, the CBSI can be used to conduct similar analysis on current or competing field artillery systems.

The Markov Model for *Crusader* and its supporting models provide a transparent, low-resolution, analytical representation of the life and productivity of a *Crusader*. The sensitivity of *Crusader* effectiveness to parametric changes in *Crusader* capabilities may be measured by several operationally significant results of calculations using the model. The user of the model simply provides the required input to the CBSI, the CBSI returns results with minimal computing effort.

I. INTRODUCTION

A. BACKGROUND

The field artillery is the "King of Battle" for many reasons. Field artillery fires destroyed the Confederate Army at Gettysburg and destroyed the will of Americans to fight at Kah Sahn, Vietnam. Field artillery fires not only destroy the enemy but also neutralize or suppress them so maneuver forces fight with great advantage. The field artillery system can fire at day and night, with no weather constraint. Few other weapons on the battlefield contribute as much as the field artillery system. [Ref. 1]

There are many reasons to modernize the field artillery system. In the Persian Gulf War, the speed of the M1 tanks and M2 Bradley fighting vehicles was limited to that of the slower M109/M992 field artillery system. The longer ranges of enemy artillery provide a significant counterfire advantage over the U.S. artillery system. The current system lacks the digitization and automation necessary to leverage technology against a numerically superior enemy force. The U.S. Army recognizes the need for a modernized field artillery weapon system. [Ref. 2]

The *Crusader* and its companion vehicle, the *Future Armored Resupply Vehicle* (FARV) are being developed for the U.S. Army to replace the current systems by the year 2006. The *Crusader*/FARV will be designed with significantly increased capabilities when compared to the current field artillery system. The *Crusader*'s range will be 10-15 kilometers greater than that of the current system. Unlike the current system, the *Crusader*/FARV will be required to move at speeds equal to the M1 tank and M2 fighting vehicles. Advanced digital communications and information systems will increase the survivability of the *Crusader*/FARV over current systems by providing greater battlefield situational awareness. The advanced capabilities of the *Crusader*/FARV over existing systems will result in a modern field artillery system which will play a decisive role in future battles. [Ref. 2]

The Test and Evaluation Master Plan (TEMP) details the program of Developmental Test and Evaluation (DT&E) and Operational Test and Evaluation (OT&E) to be conducted on the *Crusader*/FARV. Results of the initial DT&E/OT&E were provided in the first Cost and Operational Effectiveness Analysis (COEA). The COEA identified *Crusader* mobility, range, rate

of fire, and the FARV ammunition transfer rate (ATR), among others, as major drivers of the *Crusader*/FARV combat effectiveness results. Additionally, the COEA provided a comparison of combat results for a *Crusader*/FARV meeting development requirements and a *Crusader*/FARV with current system characteristics. However, the COEA did not provide sensitivity analyses to show incremental changes in combat effectiveness for parametric changes in the *Crusader* capabilities. [Ref. 3]

B. PROBLEM DEFINITION

The updated TEMP requires the next COEA to include sensitivity analysis of combat effectiveness for parametric changes to *Crusader*/FARV capabilities. The requirement calls for models to be constructed to perform the desired sensitivity analysis. The models must provide closed-form analytical results or provide for analysis on a user-interface spreadsheet. The intent of the requirement is to construct simple transparent model(s) to determine the leverage each capability has on *Crusader*/FARV combat effectiveness. [Ref. 3]

C. PURPOSE AND OVERVIEW

The purpose of this thesis is to present a transparent, low-resolution analytical model which provides for sensitivity analysis of *Crusader* effectiveness to parametric changes in *Crusader* capabilities. The specific objectives are to provide the following:

1. A description of finite-state, discrete-time Markov models developed to conduct parametric sensitivity analysis for the *Crusader*.
2. A description of the user-interface spreadsheet developed for the implementation of the model.
3. A demonstration of the models' use to conduct sensitivity analysis on the spreadsheet.

The motivation for this study is model presentation and demonstration.

The Markov models were developed in four phases. The Markov Model for *Crusader* (MMC) was developed first. Computational results of the MMC provide the values for parametric sensitivity analysis of *Crusader* effectiveness. The Markov Model for *Crusader* Availability (MMCA) was developed second. The resultant long-run availability of a *Crusader* pair serves as input to the MMC. A series of simple models to describe some of the input

parameters of the MMC and MMCA were developed third. The purpose of these models is to increase the fidelity in the computational results of the MMC by increasing the number of *Crusader* characteristics included explicitly in the models. The *Crusader* Battalion Spreadsheet Implementation (CBSI) of the Markov models was developed last. The CBSI provides a computational tool for the Markov models.

This thesis is organized in a manner similar to the development of the models. Chapter II will describe the *Crusader*/FARV and provide a summary of the operational and organizational concepts of their employment by the U.S. Army. Chapter III presents the formulation of the Markov Model for *Crusader*. Chapter IV presents a formulation of the Markov Model for *Crusader* Availability. Detailed numerical examples of the computations for the models are included in appendices to the chapters. Chapter V presents the models for some of the parameters of the MMC and MMCA. Chapter VI includes a description of the *Crusader* Battalion Spreadsheet Implementation (CBSI) of the models. This chapter includes a verification of the spreadsheet implementation and a demonstration of its use to conduct sensitivity analysis. Chapter VII provides conclusions, recommendations for model enhancements, and a brief summary.

II. *CRUSADER* AND FARV DESCRIPTION, ORGANIZATION, AND OPERATIONS

A. *CRUSADER* AND FARV DESCRIPTION

The *Crusader* is a 155mm, self-propelled howitzer that is being developed to provide indirect fires to support maneuver forces on the future battlefield for the U.S. Army. The *Future Armored Resupply Vehicle* (FARV) is the companion ammunition and fuel resupply vehicle for the *Crusader*. The *Crusader* and FARV "system" will provide maneuver forces a lethal, mobile, survivable, and sustainable indirect fire support asset.

The *Crusader* will be designed to provide indirect fires for maneuver forces that are more timely and at greater ranges and accuracy than those furnished by the current field artillery weapon. The maximum range of the *Crusader* will increase the area that a field artillery unit can cover forward of its positions by up to 170% over the current system. Additionally, the *Crusader* maximum range will create a range parity with enemy artillery, thereby reducing the counterfire threat and making the *Crusader* more survivable on the battlefield than the current system. The *Crusader*'s increased rate of fire will enable two *Crusaders* to provide the same number of rounds on target simultaneously, as can a platoon of the current system. First round accuracy of the *Crusader* will be improved by an on-board Global Positioning System (GPS) which will provide continuous information on position location for the *Crusader*. The *Crusader*'s ability to support maneuver forces will be limited only by an inability to provide it fuel and ammunition. [Ref. 2]

The FARV will be designed to provide ammunition and fuel to the *Crusader* where and when it needs it. The increased rate at which ammunition and fuel will be transferred from the FARV to the *Crusader* will enhance the ability of the *Crusader* to shoot and move. The ammunition and fuel resupply system on the FARV will be fully automated which will reduce the personnel required to operate it by two over the current resupply vehicle. The increased payload of the FARV, compared to the current resupply vehicle, will provide for more rounds to be delivered to the *Crusader*. The FARV should significantly improve the *Crusader* battalion's ammunition and fuel resupply responsiveness. [Ref. 2]

Arguably, the *Crusader*/FARV's increased mobility will be the most significant improvement over the current field artillery system. Increased vehicle control and locomotion

capabilities will provide both systems the mobility to maneuver with forces equipped with the M1A2 tanks and M2/M3 infantry fighting vehicles. Improved automotive systems will significantly increase the Reliability, Availability, and Maintainability (RAM) of the *Crusader*/FARV. Additionally, the improved automotive systems should reduce the *Crusader*/FARV vulnerability to threat engagement by providing greater mobility. Appendix A provides a summary of some of the important performance characteristics of the *Crusader*/FARV compared to the current M109M992-series field artillery systems. [Ref. 2]

B. ORGANIZATION

The *Crusader*/FARV battalion organization is expected to be similar to that of the current M109/992-series artillery battalions. Each battalion will have a headquarters battery, service battery, and three identical firing batteries. Each firing battery will consist of a headquarters section, an ammunition supply section, and two identical firing platoons. Each platoon will have a headquarters element, a Platoon Operations Center (POC), four *Crusaders*, and four FARVs. [Ref. 2]

C. OPERATIONS

The purpose of the field artillery is to provide indirect fire support to the maneuver forces. To accomplish this purpose, a command relationship is established between field artillery organizations and a maneuver force headquarters. The maneuver force commander then assigns each field artillery battalion a tactical mission. A field artillery tactical mission establishes a relationship between the battalion and a maneuver unit, or another field artillery unit. The four standard field artillery tactical missions are *direct support*, *reinforcing*, *general support reinforcing*, and *general support*. A detailed definition of these missions is included in Reference 4. Assignment of each of these missions describes in detail the inherent responsibilities of the field artillery unit with respect to priority of calls for fire, zone of fire, requirements for fire support teams, liaison requirements, communications links, positioning authority, and fire planning. [Ref. 2]

Regardless of which tactical mission is assigned to individual field artillery battalions, the

field artillery system must be able to perform seven basic tasks to accomplish the purpose of providing indirect fire support to the maneuver forces. Those seven basic tasks are *coordinate fire support, acquire targets, deliver field artillery fires, communicate, move, maintain and resupply, and survive*. These seven tasks are accomplished by a variety of personnel and equipment included in the battalion organization. A detailed description of how the *Crusader*/FARV contributes to the accomplishment of the seven tasks and the operations in which they are performed is described in Reference 2. For the purpose of this thesis, a brief description of the deployment, resupply, and firing concepts follows. [Ref. 2]

1. Deployment Concept

The *Crusader* batteries may be deployed in one of three ways: 1) by platoon; 2) by pairs; 3) by individual sections. A *Crusader* platoon will occupy positions within a 1 to 2 kilometers (km) diameter platoon position area. The position area will be 5 to 10 km behind the forward line of friendly troops. Within the platoon position area, the *Crusaders* will typically operate in pairs. The *Crusaders* in a pair will occupy positions 500-750 meters (m) apart, but within sight of each other in order to provide security for one another. Within the platoon position area, the *Crusader* pairs will move (500-750 m), emplace, fire, displace to another position. At any time in position, the *Crusaders* will also be resupplied, perform maintenance, and conduct other activities. The Platoon Operations Centers (POCs) manage movement and operations within the platoon position areas. The platoon position areas will be moved periodically with the maneuver forces so that *Crusaders* remain within 5-10 km of the forward line of friendly troops. The battery commander manages movement of the position areas. The choice of employment technique generally depends on the mission of the forces, the enemy, and the terrain, among other things. [Ref. 2]

2. Resupply Concept

The focal point for resupply operations will be the battalion Logistics Resupply Points (LRPs). The LRPs will be located 2-15 km behind the platoon position areas. At the LRP, each FARV will be reloaded with ammunition and fuel to meet the specific needs of the *Crusader* platoon that it supports. Once uploaded the FARV returns to a loiter position typically 1-2 km outside the platoon position area to await resupply instructions. [Ref. 2]

The counter-fire threat will dictate whether the FARVs remain in the loiter position or

move to their assigned platoon position area while awaiting orders to resupply *Crusaders*. The POC will dispatch a FARV to the *Crusaders* for resupply. When resupply is complete the FARV will return to its position in the platoon area, the loiter position, resupply another *Crusader* or return to the LRP to be resupplied itself. [Ref. 2]

3. Firing Concept

The firing concept for artillery consist of three components: 1) fire planning; 2) technical fire direction; 3) tactical fire direction. Fire planning is the continual process of selecting targets on the battlefield on which fires are prearranged to support particular phases of the maneuver commander's battle plan. The purpose of fire planning is to optimize the use of fire support systems to integrate and synchronize their use at the appropriate time on the battlefield. Technical fire direction is the computation of firing data for the weapons systems. Primarily, the computations are performed with computers in the POC then transmitted to the firing system digitally. However, the *Crusader* has an on-board computer which will compute firing data as required to execute sensor-to-shooter missions. Technical firing computations include such parameters as the weather, firing element location, target location, and munitions to be fired, among other things. The result of the computations is the physical aimpoint settings on the particular firing system. Tactical fire direction is the process of analyzing fire requests to determine an appropriate method of attack. Tactical fire direction considers the target characteristics, battlefield geography, the fire control measures in effect, weapon's effectiveness data, and the maneuver commander's attack criteria. The result of tactical fire direction is a fire order that prescribes the method of fire, ammunition type and expenditures, units to fire, and time to attack. [Ref. 4]

An important product of fire planning is the Attack Guidance Matrix (AGM). The AGM specifies the standard fire orders and the priority in which particular target types are attacked. The battalion fire direction center and the POCs use the AGM to expedite formulation of the fire order for a particular mission. An example AGM is in Figure 1.

	Attack	Guidance	Matrix	
Priority	**Target Description**	**Method of Engagement**	**Munitions**	**Special Instructions**
1	MRLs	Platoon, 4 rounds	DPICM	
2	Artillery	Platoon, 4 rounds	DPICM	
3	ADA	Pair, 4 rounds	HE/VT	On-order priority 1
4	OP/CP	Pair, 4 rounds	DPICM	
5	MVR/Tank	Battery, 4 rounds	DPICM	
6	MVR/APC	Battery, 4 rounds	DPICM	
7	Other	Pair, 4 rounds	DPICM	

Figure 1. Attack Guidance Matrix

III. THE MARKOV MODEL FOR *CRUSADER*

This chapter describes a finite-state, discrete-time Markov Model for *Crusader* (MMC). The material in this section is taken from Reference 8. Computational results of the MMC provide the values for the parametric sensitivity analysis of *Crusader* effectiveness.

A. MODEL DESCRIPTION

The nature of *Crusader* operations lends itself to low-resolution analytical modeling by use of a finite-state, discrete-time Markov chain. Such a model assumes that a *Crusader* pair will be in any one of a finite number of states (e.g., firing, moving, resupplying, etc.) during discrete time periods. The probability distribution of the state of a *Crusader* pair at time $t + 1$ depends only on the state at time t and does not depend on the states previously occupied by the *Crusader*. A finite-state, discrete-time Markov chain model describes the stochastic nature of *Crusader* operations. [Ref. 5]

The MMC presented in this chapter provides for analysis of the productivity and lifetime of a *Crusader* pair; productivity is measured in the number of fire missions executed by *Crusaders*. The model incorporates the arrival of different fire mission types, *Crusader* availability, and death of a *Crusader* pair as a result of enemy counterfire. During its lifetime, a *Crusader* pair will wait for fire missions, execute survivability moves, conduct resupply and administrative activities, and execute fire missions until the pair is killed.

Sensitivity to the model parameters may be measured by several results of calculations using the MMC: 1) the expected time until a particular number of *Crusaders* is killed; 2) the expected number of fire missions executed by the *Crusader* pairs; 3) the expected number of fire missions not executed as a result of *Crusaders* pairs being killed by the enemy during firing; 4) the expected number of fire missions executed, and the expected number lost by *Crusader* pairs at any finite time t; 5) the expected number and variance of the number of *Crusader* pairs surviving at any finite time t. The variance, computed in the last result, is especially important because it can affect the number of

operational tests that will need to be conducted to assess the performance of the *Crusader*. It is seen that the MMC provides for a variety of significant results.

B. MODEL FORMULATION

1. Parameters

The formulation of the MMC requires the specification of the following parameters:

Define

$H(t)$: the number of *Crusader* pairs alive at time t ($t = 0, h, 2h, \ldots$).

Let

$p_m(t)$	=	the probability a fire mission arrives in the time interval ($t, t + h$];
r_c	=	the probability a fire mission is of type c ($c = 1, \ldots, C$), requiring c *Crusader* pairs to fire;
$\alpha(H(t); t)$	=	the probability an alive *Crusader* pair is available for a fire mission in ($t, t + h$];
$h_k(c)$	=	the probability a *Crusader* pair engaged in a fire mission of type c is killed.

A detailed description of the model parameters is included in Appendix B, Section A. The availability parameter, $\alpha(H(t); t)$, is computed as described in Chapter IV. A model for the parameter, $h_k(c)$, is presented in Chapter V, Section B.

2. Assumptions

The following assumptions are necessary:

- The number of *Crusader* pairs that are available for a fire mission during the interval ($t, t + h$] has a binomial distribution with $H(t)$ trials and probability of success $\alpha(H(t); t)$. The current version of the MMC has $\alpha(H(t); t)$ constant. Future versions of this model will make this parameter time-dependent.
- A fire mission is lost (rejected) if the required number of *Crusader* pairs for the particular type mission is not available during the time increment $(t, t + h]$.
- If c *Crusader* pairs are engaged in a fire mission of type c, the number of pairs that are killed by enemy counterfire has a binomial distribution with c trials and probability of success $h_k(c)$. Note that this parameter must be specified for this model. In more comprehensive models, it will depend on the quantity and quality of the enemy counterfire force.

- *Crusader* pairs are considered to conduct all activities together.
- *Crusader* pairs can only be killed when executing a fire mission.
- The probability a *Crusader* transitions from state i to state j remains stationary over time. In this model, the state space is the number of *Crusader* pairs alive at time t ($t = 0, h, 2h, \ldots$).
- The discrete time increment in the model, h, is 5 minutes. This time length is arbitrary. It is assumed a *Crusader* pair executes a fire mission and a survivability move in 5 minutes.
- The probability that a fire mission arrives during time h in this model is independent of time. Future versions of this model will make this parameter, $p_m(t)$, time-dependent.

3. Formulation

The number of *Crusader* pairs available at time t, $\{ H(t); t = 0, h, 2h, \ldots \}$ is a finite-state, discrete-time Markov chain with stationary transition matrices of the form:

$$P(t) = \left[\begin{array}{c|cccc} 1 & 0 & 0 & \cdots & 0 \\ \hline q_1(t) & Q_{11}(t) & Q_{12}(t) & \cdots & Q_{1H(0)}(t) \\ q_2(t) & Q_{21}(t) & Q_{22}(t) & \cdots & Q_{2H(0)}(t) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ q_{H(0)}(t) & Q_{H(0)1}(t) & Q_{H(0)2}(t) & \cdots & Q_{H(0)H(0)}(t) \end{array}\right] \tag{3.1}$$

for $j > i$

$$Q_{ij}(t) = 0 \tag{3.2}$$

for $1 \le j < i$

$$Q_{ij}(t) = \underbrace{p_m(t)}_{\text{prob mission arrives}} \underbrace{\sum_{c=(i-j)}^{C} r_c}_{\text{prob the mission requires at least } i-j \text{ pairs}} \underbrace{\sum_{l=c}^{i} \binom{i}{l} \alpha(i;t)^l \left(1-\alpha(i;t)\right)^{(i-l)}}_{\text{prob at least the number of Crusader pairs needed for the mission are available}} \underbrace{\binom{c}{i-j} h_k(c)^{i-j} \left[1-h_k(c)\right]^{c-(i-j)}}_{\text{prob } i-j \text{ Crusader pairs executing the mission are killed}} \tag{3.3}$$

and

$$q_i(t) = 1 - \sum_{j=1}^{H(0)} Q_{ij}(t) \tag{3.4}$$

where $i, j = 1, 2, \ldots H(0)$, the number of *Crusader* pairs available at time 0. A three step procedure to compute the transition probability matrix, $P(t)$, along with a numerical example, appears in Appendix C. Details of the computations using the MMC, to include a numerical example, can be found in Appendix D.

IV. THE MARKOV MODEL FOR *CRUSADER* AVAILABILITY

This chapter describes the Markov Model for *Crusader* Availability (MMCA). The material in this chapter is taken from Reference 8. A result of the model, the long-run proportion of time a *Crusader* pair is available to conduct fire missions, is used in the MMC for the value of the parameter, $\alpha(H(t); t)$. Additionally, the results of the model, themselves, provide probabilistic information about the proportion of time a *Crusader* pair is available, conducting ammunition resupply, or under repair.

A. PARAMETERS

Let

$X(t)$ = be the state of the *Crusader* pair at time t ($t = 0, h, 2h, \ldots$).

The successive states visited in the discrete-time Markov chain are:

A	=	*Crusader* pair is available to execute a fire mission;
B	=	*Crusader* pair is being resupplied with ammunition;
R	=	*Crusader* pair is awaiting repair or under repair.

The following parameters must be specified:

$p_m(t)$	=	the probability a mission arrives in a time interval of length $(t, t+h]$;
y	=	the number of *Crusader* pairs alive;
p_b	=	the probability a *Crusader* needs resupply in a time interval of length $(t, t+h]$;
p_{ba}	=	the probability a *Crusader* pair completes resupply and returns to the available state in a time interval of length $(t, t+h]$;
p_r	=	the probability a *Crusader* pair needs repair in a time interval of length $(t, t+h]$;
p_{ra}	=	the probability a *Crusader* pair completes repair and returns to the available state in a time interval of length $(t, t+h]$;
r_c	=	the probability a fire mission is of type c; requiring c *Crusader* pairs to execute;

A detailed definition of the parameters is included in Section B of Appendix B. Models for some of the parameters are presented in Chapter V. These models explicitly account for several engineering characteristics of the *Crusader*/FARV.

B. ASSUMPTIONS

The following assumptions are necessary:

- The discrete time increment in the model, h, is 5 minutes. This time length is arbitrary. It is assumed a *Crusader* pair executes a fire mission and a survivability move in 5 minutes.
- *Crusader* pairs are not subject to being killed in this supporting model. *Crusaders* are subject to death in the MMC.
- *Crusader* pairs are equally likely to be assigned a fire mission requiring c *Crusader* pairs with probability of c/y if $c \leq y$; otherwise, the mission is lost.
- The FARV is always available to resupply a *Crusader* pair.
- The probability that a fire mission arrives during time h in this model is independent of time. Future versions of this model will make this parameter, $p_m(t)$, time-dependent.

C. MODEL FORMULATION

The transition probability matrix for MMCA, $\overline{P}(y)$, is of the form:

$$\overline{P}(y) = \begin{array}{c} A \\ B \\ R \end{array} \begin{bmatrix} 1 - p_m \sum_{c=1}^{C} r_c f\left(\frac{c}{y}\right)[p_b + p_r] & p_m \sum_{c=1}^{C} r_c f\left(\frac{c}{y}\right) p_b & p_m \sum_{c=1}^{C} r_c f\left(\frac{c}{y}\right) p_r \\ p_{ba} & 1 - p_{ba} & 0 \\ p_{ra} & 0 & 1 - p_{ra} \end{bmatrix} \tag{4.1}$$

where

$$f\left(\frac{c}{y}\right) = \begin{cases} \frac{c}{y} & \text{if } \frac{c}{y} \leq 1 \\ 0 & \text{if } \frac{c}{y} > 1 \end{cases} \tag{4.2}$$

Given the number of *Crusader* pairs alive is y, the probability a *Crusader* pair is available to conduct fire missions is $\alpha(y) = \lim_{n \to \infty} \overline{P}(y)^n_{AA}$. Details of the computations with a numerical example can be found in Appendix E.

V. ENGINEERING PARAMETRIC MODELS OF *CRUSADER* MARKOV CHAIN PARAMETERS

The usefulness of the MMCA and, ultimately, the MMC is measured by their capability to explicitly model engineering characteristics of the *Crusader*/FARV. This chapter presents a series of simple models for some of the parameters of the MMCA and MMC. The material in this chapter is taken from Reference 9. These models explicitly account for several engineering characteristics of the *Crusader*/FARV and provide for the desired sensitivity analysis using the Markov models. A detailed definition of the parameters and the engineering characteristics which describe them is included in Section C of Appendix B. Only the models are presented in this chapter, first, for parameters of the MMCA and second, for a parameter of the MMC.

A. MODELS FOR THE MMCA PARAMETERS

1. Model for the Probability a *Crusader* Pair Requires Ammunition Resupply (p_b)

Let

n_f	=	average number of rounds (projectiles) fired per mission by a *Crusader* pair;
N_s	=	number of missions fired before survivability move by a *Crusader* pair;
L_R	=	maximum number of rounds carried by a *Crusader*;
α_R	=	percentage of L_R on-hand when ammunition resupply is executed.

Let the number of missions fired between ammunition resupply equal:

$$N_{repl} = \left\lfloor \left(\frac{\text{number of rounds fired before resupply}}{\text{average number of rounds fired before a survivability move}} \right) \right\rfloor = \left\lfloor \left(\frac{\alpha_R L_R}{n_f N_s} \right) \right\rfloor \tag{5.1}$$

where $\lfloor \ \rfloor$ represents the largest integer less than or equal to the value. Then the probability a *Crusader* pair requires resupply in a time interval of length (t, t + h] is $p_b = \min(1 / N_{repl}, 1)$.

2. Model for the Probability a *Crusader* Pair Completes Ammunition Resupply (p_{ba})

Let

ρ = the ammunition transfer rate from the FARV to the *Crusader* (in units of $1/h$).

Let the time (in units of h) to resupply a *Crusader* pair, T_R, equal

$$2\alpha_R L_R = \rho T_R \Rightarrow T_R = \left(\frac{\text{number of rounds transferred to two Crusaders}}{\text{ammunition transfer rate}} \right) = \left(\frac{2\alpha_R L_R}{\rho} \right). \tag{5.2}$$

Then the probability a *Crusader* pair completes ammunition resupply in a time interval of length (t, t + h] is: $p_{ba} = \min(1 / T_R, 1)$.

3. Model for the Probability a *Crusader* Pair Requires Repair (p_r)

Let

m_f = the Mean-Time-Between-Essential-Function-Failures (MTBEFF) for the *Crusader*.

Let the number of 5 minute time increments between failures equal m_f x 12 (where there are 12, 5 minute increments in one hour and m_f is given in hours). Then the probability a *Crusader* pair needs repair at the end of a 5 minute time increment is: $p_r = 1 / (m_f \text{ x } 12)$.

4. Model for the Probability a *Crusader* Completes Repair (p_{ra})

Let

m_r = the Mean-Time-to-Repair (MTTR) plus Administrative-and-Logistic-Downtime (ALDT) for the *Crusader*.

Let the number of 5 minute time increments until repair is complete equal m_r x 12 (where there are 12, 5 minute increments in one hour and m_r is given in hours). Then the

probability a *Crusader* pair completes repair at the end of a 5 minute time increment is: $p_{ra} = 1/(m_r \text{ x } 12)$.

B. MODEL FOR THE PROBABILITY A *CRUSADER* PAIR IS KILLED AS A FUNCTION OF FIRING PARAMETERS

Assume that if c *Crusader* pairs are engaged in a fire mission of type c, the number of pairs that are killed has a binomial distribution with c trials and probability of success $h_k(c)$.

Compute the probability of success as follows:

Let

p_{acq}	=	probability *Crusaders* are detected by the enemy as a result of firing;
θ	=	*Crusader* rate of fire in units of rounds per minute;
n_f	=	average number of rounds (projectiles) fired per mission by a *Crusader* pair;
t_f	=	average time to fire a mission is n_f / θ in units of minutes;
N_s	=	number of missions fired before survivability move by a *Crusader* pair;
C_{fp}	=	*Crusader* counterfire footprint;
v_s	=	scoot velocity in units of meters per second;
$T_E(R)$	=	median time for enemy artillery to engage a *Crusader* after acquiring a *Crusader* in units of h minutes. The median time to engage is used because it is known data from previous *Crusader* analysis. [Ref. 2]

A detailed description of the parameters appears in Section C of Appendix B.

First, the probability enemy sensors acquire a *Crusader* pair during firing is:

$$1-\left(1-p_{acq}\right)^{2N_s} \equiv \alpha_{RAB} . \tag{5.3}$$

This models the number of failures before the first successful acquisition of a *Crusader* as a result of firing. The exponent, $2N_s$, accounts for the fact that each *Crusader* in a pair may be acquired independently during a fire mission. This model is a consequence of the assumption that the acquisitions of the *Crusader* pairs are independent and identically distributed Bernoulli trials with probability of success: p_{acq}.

Second, the amount of time, in minutes, required for a *Crusader* pair to fire then move out of the counterfire footprint, T_{ss}, is:

$$\underbrace{(t_f)N_s}_{\text{time to fire mission}} + \underbrace{\frac{C_{fp}}{v_s}}_{\text{time to move out of footprint}} = T_{ss} \tag{5.4}$$

Third, assume the time, in minutes, required for the enemy to engage *Crusaders* has an exponential distribution with mean $1 / \mu_E$. Compute μ_E as follows:

$$1 - e^{-\mu_E T_E(R)} = .5 \tag{5.5}$$

then

$$\mu_E = (1 / T_E(R))(\ln 2) \tag{5.6}$$

Therefore, the probability a *Crusader* pair is killed by counterfire is:

$$\alpha_{RAB} P\{S_E < T_{SS}\} = \left(1 - e^{-\mu_E T_{SS}}\right)\alpha_{RAB} \tag{5.7}$$

where S_E is an exponential random variable having mean $1 / \mu_E$.

For a fire mission of type *c* requiring *c Crusader* pairs to fire, the probability of success in the binomial distribution for the number of *Crusader* pairs killed is therefore:

$$h_K(c) = \alpha_{RAB}\left(1 - e^{-\mu_E T_{SS}}\right) \tag{5.8}$$

C. NUMERICAL EXAMPLES

1. Input

The following is input for the examples:

$H(0)$	=	3 *Crusader* pairs;
$p_m(t)$	=	.3 ;
r_1	=	.5 ;
r_2	=	.5 ;
v_s	=	8.3 meters per second;
C_{fp}	=	750 meters;
$T_E(R)$	=	10 minutes;
n_f	=	4 rounds per mission;
θ	=	10 rounds per minute;

N_s	=	1 mission before survivability move;
α_R	=	.6 ;
ρ	=	5 rounds per minute;
m_f	=	34 hours;
m_r	=	6.45 hours;
p_{acq}	=	.7 ;
L_R	=	60 rounds.

A detailed definition of the input parameters is in Appendix B.

2. Probability a *Crusader* Pair Requires Ammunition Resupply (p_b)

$$N_{repl} = \left(\frac{.6(60)}{4(1)}\right) = 9$$

$$p_b = 1/9 = .1111$$

3. Probability a *Crusader* Pair Completes Ammunition Resupply (p_{ba})

$$T_R = \left(\frac{2(.6)60}{5}\right) = 14.4 \text{ minutes}$$

14.4 minutes is 14.4/5 = 2.88 five minute time intervals

$$p_{ba} = 1/2.88 = .34722$$

4. Probability a *Crusader* Pair Requires Repair (p_r)

$$m_f = 34 \text{ hours}$$

$$p_r = 1/(34 \text{ x } 12) = .00245$$

5. Probability a *Crusader* Pair Completes Repair (p_{ra})

$$m_r = 6.45 \text{ hours}$$

$$p_{ra} = 1/(6.45 \text{ x } 12) = .01292$$

6. Probability a *Crusader* Pair is Killed as a Result of Firing

$$\alpha_{RAB} = 1-(1-.7)^{2(1)} = .91$$

$$T_{ss} = \left(\frac{4}{10}\right)(1) + \frac{750}{8.3(60\,\text{sec})} = 1.906 \text{ minutes}$$

$$\mu_E = (1/10)(\ln 2) = .06931 \text{ minutes}^{-1}$$

The probability of success in the binomial distribution for the number of *Crusader* pairs killed is:

$$h_K(c) = .91\left(1 - e^{-.06931(1.906)}\right) = .11262 .$$

VI. *CRUSADER* BATTALION SPREADSHEET IMPLEMENTATION

This chapter presents the *Crusader* Battalion spreadsheet Implementation (CBSI). The spreadsheet serves as the user-interface, computational tool for the MMC and MMCA. The user-interface spreadsheet is described first. A simple verification of the spreadsheet is included second. The results of the spreadsheet are checked for reasonable trends. A demonstration of the use of the spreadsheet model for parametric sensitivity analysis of *Crusader* effectiveness is included third. A demonstration of the use of the spreadsheet model for comparative analysis with the *Paladin* is included last.

A. DESCRIPTION

The CBSI was constructed on a Microsoft *Excel V7.0* spreadsheet. [Ref. 10] The Microsoft *Visual Basic* programming language [Ref. 11] was used for particular operations within the spreadsheet. The user of the spreadsheet provides the input data and selects a computational mode; the spreadsheet calculates and displays the results of the MMC and the MMCA. Instructional documentation for the CBSI software is included in Appendix F.

1. Input

The input to the spreadsheet is described in detail in Appendix B. The user must enter the data in appropriate unit (e.g., minutes, rounds per minute, etc.) in the spreadsheet. The spreadsheet then converts the input data to units of 5 minute intervals. For MMC finite-time results, the user must select the finite-time computational mode and then enter the number of battle hours for which results are desired. The input page is locked so that calculations do not occur until an appropriate computational mode is selected by the user. An example on the input page for the CBSI appears in Figure 2. The input data which appear in Figure 2 serve as the "base case" data for analysis in this chapter. The "base case" data are explained in Section B.

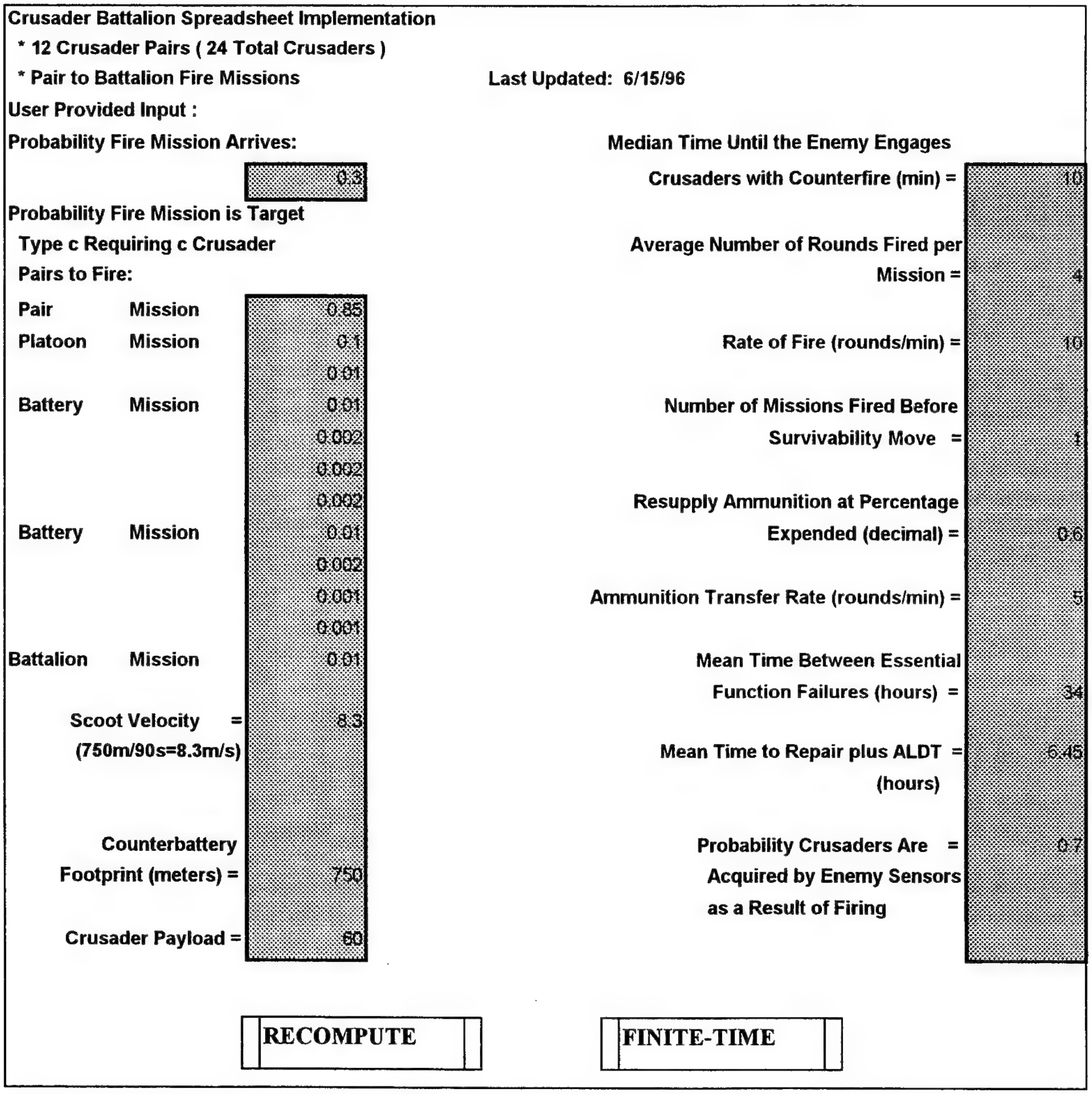

Figure 2. CBSI Base Case Data Input Page

2. Computations

The computational code in the CBSI is accessible for read-only use. A password will be provided to the user which allows access to the code so that it can be modified for any reason. The "recompute" button invokes the first passage time calculations using formulas (D.1) - (D.11) of Appendix D. The "finite-time" button invokes the calculations using formulas (D.12) - (D.19) of Appendix D.

Intermediate results for the MMC and MMCA are also accessible for examination in the spreadsheet model. For example, the resultant matrices of the three-step procedure

for computing the transition probability matrix of the MMC presented in Appendix C are displayed for the user. An example of the CBSI page for some of the intermediate computations of the "base case" input data is included in Appendix G.

The finite-time calculations for the MMC require the most computational time in the CBSI. For a 24 hour battle time, a 486DX-66 computer requires 7 minutes, 45 seconds to complete calculations and provide results. The *Visual Basic* code written to execute the finite-time calculations is displayed in Appendix H.

3. Output

The computational results of the MMC are displayed for the user after calculations are completed. An example of the CBSI results page for the "base case" input data is included in Figure 3.

Crusader Battalion Computational Results

Expected Time There Are j Crusader Pairs Alive Given Start With 12 Pairs:

(Time in Hours)

	j	Time	j	Time
	12	1.957216	6	2.149041
	11	1.906629	5	2.179115
j =	10	1.934535	4	2.209143
	9	1.961758	3	2.284881
	8	2.001763	2	2.416278
	7	2.098713	1	3.24701

Expected Time Until All Crusader Pairs are Destroyed:

26.34608	hours
1.097753	days

Expected Number of Missions Executed While Crusaders are Alive = 89.77495

Expected Number of Missions Lost While Crusaders are Alive = 5.07094

Long Run Availability Probabilities =

		Available	Resupply	Repair
	1	0.884975	0.072214	0.042811
	2	0.925687	0.046655	0.027658
	3	0.947824	0.032757	0.019419
	4	0.958943	0.025776	0.015281
Crusaders	5	0.966596	0.020971	0.012432
Alive	6	0.971719	0.017755	0.010526
	7	0.97537	0.015463	0.009167
	8	0.97692	0.01449	0.00859
	9	0.979139	0.013097	0.007764
	10	0.981038	0.011904	0.007057
	11	0.982585	0.010933	0.006482
	12	0.982534	0.010965	0.0065

Finite Time Calculations : 24 **Hours**

Expected Number of Crusader Pairs Alive 1.645128

Standard Error in Number of Crusader Pairs Alive 1.921354

Expected Total Number of Missions Executed 76.35804

Expected Total Number of Mission Lost 10.00

Figure 3. CBSI Base Case Data Output Page

B. VERIFICATION

The theoretical basis for a finite-state, discrete-time Markov chain model is well-documented and left to the interested reader for further study. The applicability of Markov theory for modeling *Crusader* operations was discussed in Section A of Chapter

III of this thesis. This section provides a brief graphical analysis to verify that the CBSI is a correct implementation of the MMC and MMCA.

The input data for this analysis and the proceeding two sections are displayed on the CBSI input page in Figure 2 of Section A. This "base case" data were taken from two sources: 1) the *Operational Test and Evaluation Concept for Crusader* (OTEC) [Ref. 2]; 2) the results of the *Crusader Timeline Model* (CTM) [Ref. 6]. Among other things, the OTEC document provides results of *Crusader* performance from OT&E/DT&E for the COEA. The CTM was developed by the Institute for Defense Analysis (IDA) for the Army Operational Test and Evaluation Division. The CTM provides information on the amount of time a *Crusader* spends participating in various operational activities. The data were taken from these sources because they represent operational results from previous analyses of the *Crusader*. The data or results may not be sufficient for future analysis with the MMC and MMCA.

1. Sensitivity to the Probability a Fire Mission Arrives

The expected time until all *Crusaders* are killed as a result of firing should decrease as the probability of receiving a fire mission in a 5 minute period increases. This trend reflects the fact that, as *Crusaders* fire more often, they are more often subject to detection, attack, and possible attrition. Figure 4, from results of the CBSI for the "base case" data, displays an intuitively appropriate relationship between *Crusader* death and the mission arrival rate.

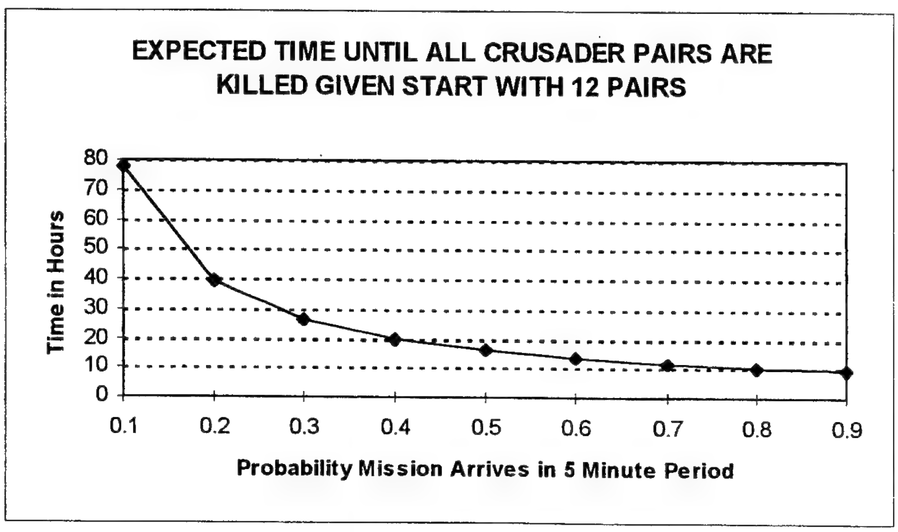

Figure 4. Sensitivity to Probability Mission Arrives

2. *Crusader* Pairs Alive Over Time

The mean number of *Crusader* pairs alive should decrease as time increases. This is because the longer *Crusaders* are in battle, the more missions they will execute, and the more often they will be subject to attrition. Figure 5, from results of the CBSI for the "base case" data, displays an intuitively appropriate relationship between *Crusaders* alive, and elapsed time in battle.

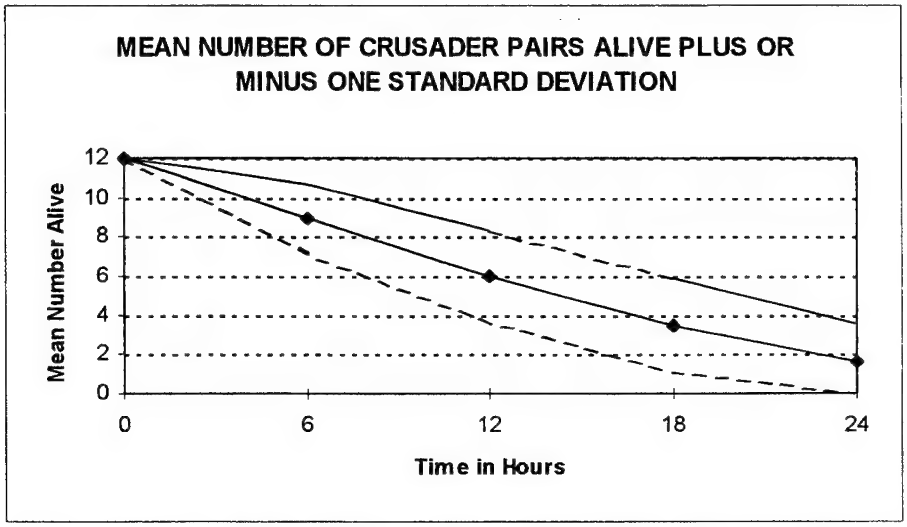

Figure 5. Mean *Crusader* Pairs Alive: Sensitivity to Time

3. Missions Executed and Lost Over Time

The number of fire missions executed and lost should increase as time increases. This is because the longer *Crusaders* are in battle, the more missions will arrive, and the more often the missions will be executed or lost. Figure 6, from results of the CBSI for the "base case" data, displays an intuitively appropriate relationship between fire missions executed and lost, and time.

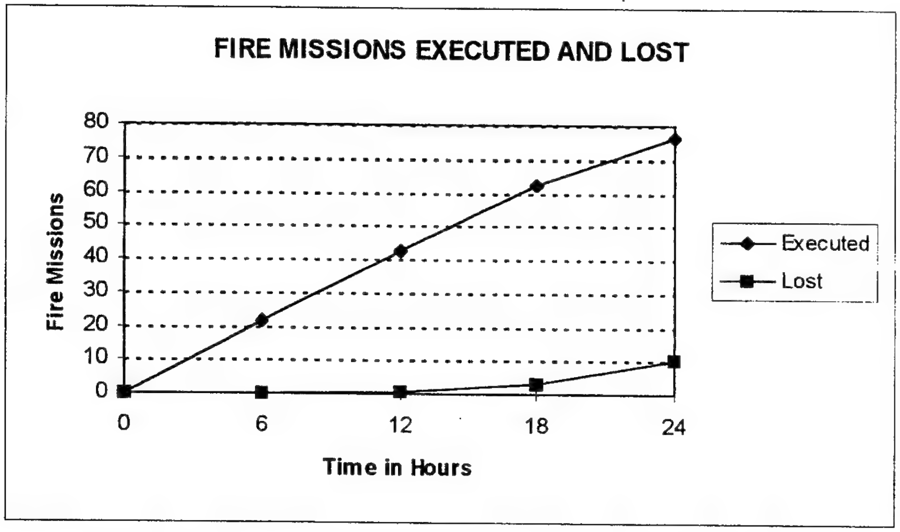

Figure 6. Fire Missions Executed and Lost: Sensitivity to Time

4. Conclusions

There are several points worth remembering when analyzing results of the CBSI. First, *Crusaders* are killed by pairs in the CBSI. It is unlikely in battle that both *Crusaders* in a pair will always be killed as a result of enemy counterfire. Second, attrition of enemy artillery is not modeled in the MMC. As enemy artillery systems are killed, fewer are available to conduct counterfire missions which may result in less *Crusader* attrition. Lastly, the intensity of battle is not modeled in the MMC. The probability of a fire mission arrival should be non-stationary over time to reflect the intensity of battle. A smaller probability of fire mission arrival during appropriate phases of the battle will result in fewer missions being fired and, therefore, less *Crusader* attrition as a result of firing. All three cases may lead to greater attrition and fewer missions fired when compared to the results of other models. For example, simulations conducted to provide the results for the COEA modeled individual *Crusaders*, enemy attrition, and the intensity of battle and, therefore, probably provide results with less *Crusader* attrition and more fire missions executed in 24 hours than those results obtained from the CBSI. [Ref. 7] Notwithstanding, the relationships between specified parameters and *Crusader* effectiveness are intuitively appealing and indicate "sensible" trends.

C. SENSITIVITY DEMONSTRATION

The ultimate test of the MMC and MMCA and their spreadsheet implementation is whether or not they can be used to conduct sensitivity analysis on specified engineering parameters of the *Crusader*. This section provides examples to demonstrate the use of the CBSI for sensitivity analysis of *Crusader* effectiveness to changes in *Crusader* rate of fire, scoot velocity, and FARV ammunition transfer rate (ATR). A detailed description of the engineering characteristics is included in Appendix B.

1. Rate of fire and Scoot Velocity

The rate of fire is the maximum rate at which a *Crusader* can deliver fires. An increase in the rate of fire should increase *Crusader* survivability by reducing the time spent firing and thereby decrease the *Crusader's* vulnerability to enemy counterfire. Figure 7, from results of the CBSI for the "base case" data, displays the relationship of rate of fire and the mean number of *Crusaders* alive after 24 hours of battle.

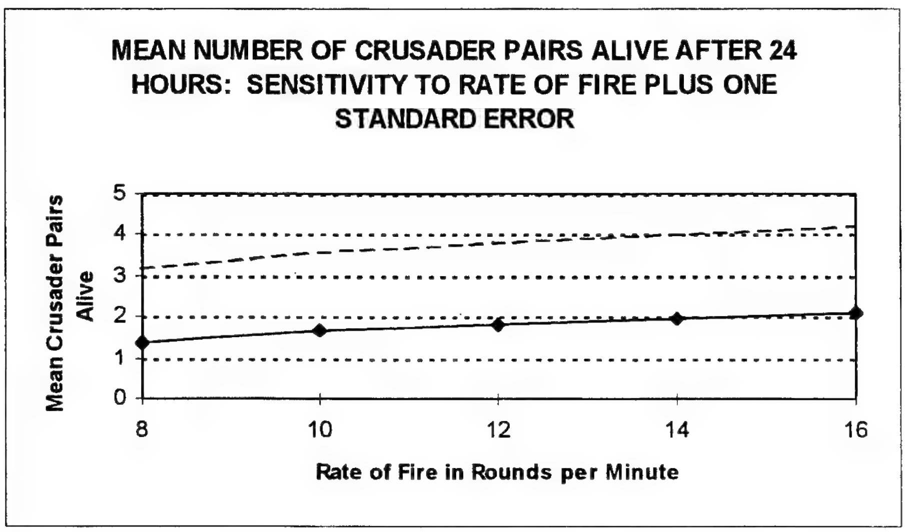

Figure 7. Mean *Crusader* Pairs Alive: Sensitivity to Rate of Fire

The *Crusader* scoot velocity is the rate at which it exits the counterfire footprint after executing a fire mission. The speed of the *Crusader*, among other things, affects the time to exit the footprint. The number of *Crusaders* alive after 24 hours should increase if the scoot velocity is increased because the *Crusader* is less susceptible to enemy counterfire. Figure 8, from results of the CBSI for the "base case" data, displays the relationship between scoot velocity and the number of *Crusader* pairs alive.

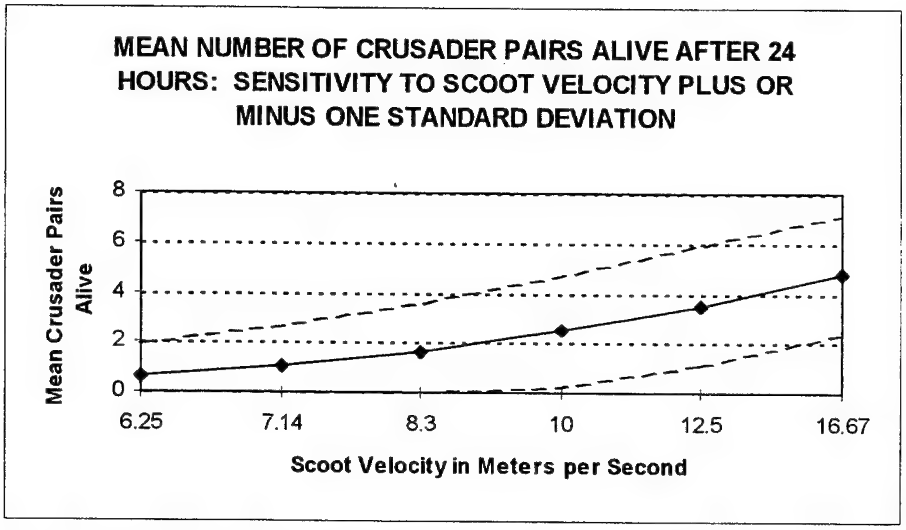

Figure 8. Mean Crusader Pairs Alive: Sensitivity to Scoot Velocity

The number of *Crusader* pairs alive after 24 hours of battle appears more sensitive to *Crusader* scoot velocity than rate of fire. Improving the scoot velocity above the ORD requirement of 8.3 meters per second (750m/90sec) to 10 meters per second (750m/75sec) increases the number of *Crusader* pairs alive by 50%. Improving the rate of fire above the ORD requirement of 10 rounds per minute to 12 rounds per minute increases the number of *Crusader* pairs alive by only 12%. Additionally, the figures display a significant increase in the number of *Crusader* pairs alive as the scoot velocity increases, while the increased *Crusader* survival as a result of rate of fire increases appears negligible. However, the rate at which *Crusader* survival improves as a function of scoot velocity will decrease as the number of rounds fired per mission increases (4 rounds per mission in this analysis). Therefore, further analysis may be required to determine the contribution of scoot velocity to *Crusader* survival.

2. FARV Ammunition Transfer Rate

The ammunition transfer rate (ATR) is the rate at which the FARV transfers ammunition to the *Crusader*. The ORD requires an ATR of 5 rounds per minute (60 rounds in 12 minutes). An increase in the ATR should increase the amount of time a *Crusader* pair is available to conduct fire missions. Figure 9, from results of the CBSI for the "base case" data, displays the relationship between ATR and *Crusader* availability.

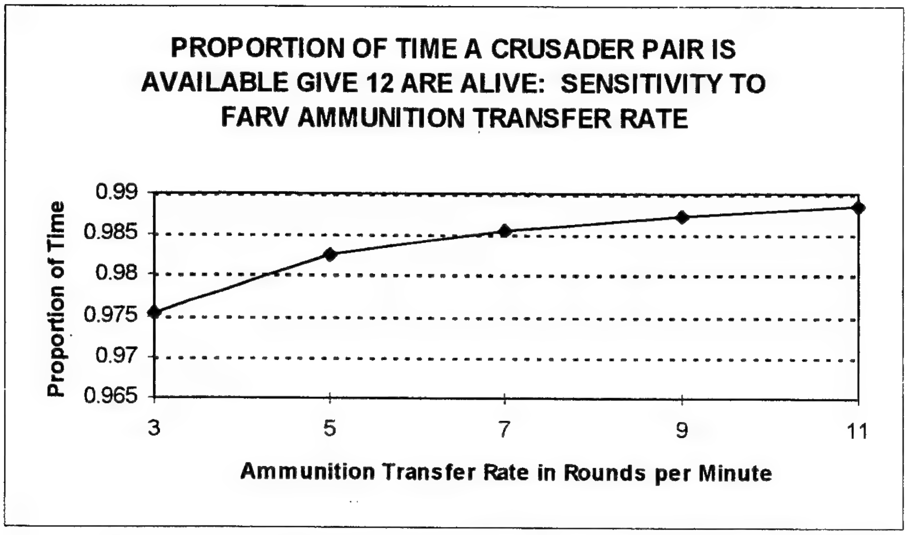

Figure 9. *Crusader* Pairs Available: Sensitivity to Ammunition Transfer Rate

While the figure displays an increase in the proportion of time a *Crusader* pair is available as a result of improving the ATR, the increase in terms of real time on the battlefield may be insignificant.

D. *CRUSADER* vs. *PALADIN* DEMONSTRATION

Certainly, the need will arise to compare the *Crusader*'s effectiveness to current or competing field artillery systems. This section provides an example to demonstrate the use of the CBSI for analyzing the *Crusader*'s effectiveness as compared to the current field artillery system: the M109A6 *Paladin*.

The input data for this demonstration reflects the differences of the two systems. The data for the *Crusader* was discussed in Section B and not changed for this demonstration. The data for the *Paladin* were also taken from the OTEC and CTM. The *Paladin* engineering characteristics data (e.g., rate of fire, ATR, scoot velocity, MTBEFF, payload, and counterfire footprint) were inputted accordingly in the CBSI for this demonstration. Additionally, the probability that an arriving target is of a particular type was changed to reflect the fact that most enemy targets will require a *Paladin* platoon fire mission. This is because of the decreased lethality and slower rate of fire of the *Paladin* as compared to the *Crusader*, which will primarily participate in pair fire missions. All other data for the demonstration (e.g., probability a fire mission arrives, number of rounds fired

per mission, probability of acquisition by enemy sensors, etc.) remains the same for the *Crusader* and the *Paladin*. The CBSI input page for the *Paladin* is included in Appendix I. The input data for the demonstration reflects the engineering and tactical deployment differences of the *Crusader* and *Paladin*.

1. *Crusader* vs. *Paladin* Pairs Alive Over Time

The mean number of *Crusader* pairs alive over time should be higher than the mean number of *Paladins* alive over time because *Crusaders* are more survivable on the battlefield as compared to *Paladins*. This superior survivability is due to, among other things, the *Crusader*'s higher scoot velocity (8.3 meters per second vs. 6.22 meters per second) and rate of fire (10 rounds per minute vs. 4 rounds per minute). Figure 10 displays an intuitively appropriate comparative relationship between *Crusaders* and *Paladins* alive, and elapsed time in battle from results of the CBSI,.

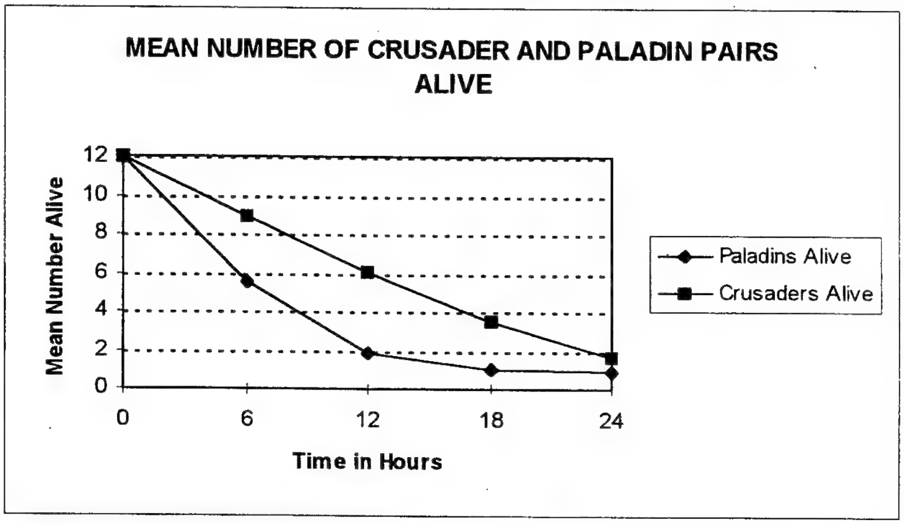

Figure 10. Mean Number of *Crusader* and *Paladin* Pairs Alive Over Time

2. *Crusader* vs. *Paladin* Fire Missions Executed and Lost Over Time

The expected number of fire missions executed by *Crusaders* should be higher than the expected number of fire missions executed by *Paladins*, while fewer fire missions should be lost by *Crusaders* than by *Paladins*. These effects are due to the fact that the mean number of *Crusaders* alive over time tends to be larger than the mean number of *Paladins* alive over time. Thus, more *Crusaders* than *Paladins* will tend to be available to execute fire missions. Figure 11, from results of the CBSI, displays an intuitively

appropriate comparative relationship between missions lost and executed, and elapsed time in battle.

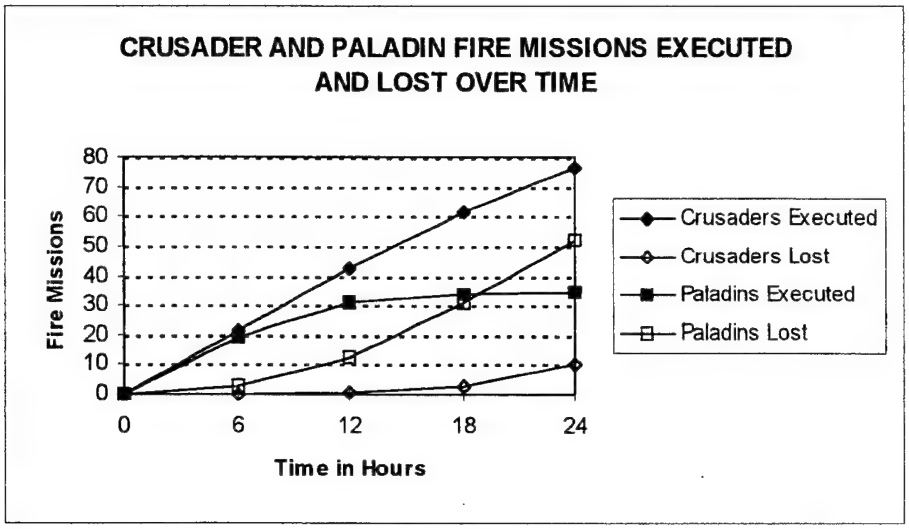

Figure 11. ***Crusader*** **and** ***Paladin*** **Missions Executed and Lost Over Time**

3. Analysis

There are several observations from the *Crusader* and *Paladin* comparison worth highlighting. First, the mean number of fire missions executed by *Crusaders* is over twice as large as the mean number of fire missions executed by *Paladins*. The *Crusaders'* higher rate of fire and greater survivability contribute to this result. Second, the mean number of fire missions not executed by *Crusaders* (10 missions lost) because they are unavailable (e.g., killed, under repair, executing resupply) is considerably smaller than the mean number of missions not executed by *Paladins* (52.2 missions lost). More important, the mean-fire-missions-executed-to-mean-fire-missions-lost ratio (mean missions executed / mean missions lost) is considerably smaller for the *Paladin* (34.20/52.2 = .65) than for the *Crusader* (76.36/10 = 7.64). The *Crusader*, on the average, executes more missions and loses fewer than the *Paladin*. These results need to be verified by results from other models. Notwithstanding, the comparative relationships are intuitively appealing and this example demonstrates the usefulness of the CBSI of the MMC for such comparative analysis.

VII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The models of this thesis lead to several conclusions. First, the Markov Model for *Crusader* (MMC) and its supporting model, the Markov Model for *Crusader* Availability (MMCA) provide a transparent, low-resolution, analytical representation of the life and productivity of a *Crusader.* Furthermore, the sensitivity of *Crusader* effectiveness to variations in the MMC parameters may be measured by operationally significant results of calculations using the model.

Second, enhancement of the Markov models by the development of a series of simple models for some of the parameters provides for the desired sensitivity analysis of *Crusader* effectiveness with respect to specified engineering characteristics. These supporting models provide the user of the MMC with the explicit capability to analyze the leverage a particular engineering parameter has on *Crusader* effectiveness.

Third, the *Crusader* Battalion Spreadsheet Implementation of the MMC and MMCA provides the computational tool to conduct the sensitivity analysis. The CBSI provides "sensible" and "explainable" results from the MMC and MMCA with minimal computational effort.

Lastly, the MMC can easily be used to conduct similar analysis on other artillery systems and in particular to compare the *Crusader* to current or competing field artillery systems. Certainly, the need will arise to compare the *Crusader*'s effectiveness to the current field artillery system, the M109A6 *Paladin.* The inputs to the models easily accommodate characteristic data of the *Paladin.* The Markov models and the CBSI provide a means to conduct alternative system effectiveness comparisons.

B. RECOMMENDATIONS

Time did not permit some desirable enhancements of the current Markov models or their spreadsheet implementation, the CBSI. The following are some enhancements proposed for further study.

One modification may be to let some of the parameters of the MMC and the MMCA be functions of time. For example, the fire mission arrivals may be modeled as a discrete-time, *non-homogeneous* Poisson process. This modification will provide for the intensity of battle to be modeled in the MMC and MMCA.

The MMCA may be improved by enlarging the state space in the model. In its current form, the resultant *Crusader* long-run availability may be too high. While this is possible given the requirements for the *Crusader*, it more likely that, in maneuver battle, other *Crusader* activities (e.g., moving) will decrease the time *Crusaders* are available to conduct fire missions. Increasing the MMCA state space will improve the resolution of the MMCA by incorporating other activities of *Crusaders* into the model.

The model for the probability with which a *Crusader* completes resupply does not include administrative time or time for the FARV to move to the *Crusader* from its current position. These times can easily be added to the parametric model and will provide a more realistic representation of the resupply operations.

The *Visual Basic* program in the CBSI should be improved to provide for a greater flexibility in the number of *Crusaders* and mission types available to the user. For instance, the need may arise to analyze a Division Artillery (3 battalions = 72 *Crusaders*). The MMC provides for analysis of any number of *Crusaders* and mission types. In its current form, the CBSI does not allow this flexibility. A maximum of 12 *Crusader* pairs (battalion) are modeled in the CBSI. This improvement will eradicate the current requirement that the 24 *Crusaders* in a battalion conduct all activities together.

The results of Chapter VI demonstrated that the Markov models' behaviors agree with intuition. However, these results cannot be considered to be a complete validation of the models. The results of the MMC need to be further compared to results of other models, and to field exercise data, if available. Additionally, the distributional assumptions for the parameter models need to be tested against data for the activities the represent. While every effort has been made, within the time available, to ensure that the models provide accurate results, further validation efforts are necessary to ensure complete confidence in their predicted outcomes.

C. SUMMARY

The U. S. Army has identified a requirement for a new field artillery weapon system to support maneuver forces in battle in the next century. The *Crusader* and its companion vehicle, the *Future Armored Resupply Vehicle* (FARV) are under development to fulfill the requirement. Continued DT&E/OT&E requires the use of transparent, low-resolution analytical modeling to conduct sensitivity analysis on specified engineering characteristics of the *Crusader*/FARV.

The purpose of this thesis has been three-fold: 1) to describe the finite-state, discrete-time Markov models developed to conduct the desired sensitivity analysis on specified engineering characteristics of a *Crusader*/FARV; 2) to describe the user-interface spreadsheet developed for the implementation of the Markov models; 3) to demonstrate the models' use to conduct the desired sensitivity analysis using results from the spreadsheet implementation of the model.

The nature of *Crusader* operations lends itself to modeling by a Markov chain, and the Markov models developed in this thesis were demonstrated to be useful. However, the results are only as good as the assumptions used to create the models and the accuracy of the values for estimated parameters. Implementing the recommended enhancements to the models should increase usefulness and validity in the results obtained from the current models. Empirical studies should be conducted to estimate the models' parameters. After doing so, the MMC and its supporting models will provide flexible, low-resolution, analytical models for the parametric sensitivity analysis of *Crusader* effectiveness.

APPENDIX A. COMPARISON OF *PALADIN*/FAASV AND *CRUSADER*/FARV OPERATIONAL REQUIREMENT CHARACTERISTICS

This appendix presents some of the critical operational requirements of the *Crusader*/FARV and the current field artillery system, the *Paladin*/FAASV. These data were taken from the Operational Requirements Document for the *Crusader*/FARV and summarized in the OTEC [Ref. 2] . Reference 2 includes a complete listing of similar data on the systems. Table 1 and 2 display some characteristic data for the systems.

Critical Characteristic	*Crusader*	*Paladin*
Maximum Range	40-50 km	30 km
Rate of Fire (burst)	10 rounds/minute	4 rounds/minute
Payload	60 rounds	39 rounds
Cross-Country Mobility	39-48 km per hour	27 km per hour
Scoot Velocity	8.3 meters/second (750m/90s)	6.22 meters/second (560m/90s)
MTBEFF	34 hours	21 hours
MTTR	2 hours	2 hours
Crew Size	3 soldiers	4 soldiers

Table 1. Operational Requirement Characteristics of the *Crusader* and *Paladin*

Critical Characteristic	FARV	FAASV
Ammunition Transfer Rate	60 rounds in 12 minutes	60 rounds in 35 minutes
Payload	130 rounds	90 rounds
Cross-Country Mobility	39-48 km per hour	31 km per hour
Crew Size	3	5

Table 2. Operational Requirement Characteristics of the FARV and FAASV

APPENDIX B. INPUT PARAMETERS DESCRIPTIONS

This appendix provides a description of the parameters used in the MMC, MMCA, and CBSI.

A. MARKOV MODEL FOR *CRUSADER* PARAMETERS

Parameters

$p_m(t)$ This parameter represents the stationary probability that an enemy target will arrive during a 5 minute time increment $(t, t+h]$. The greater the probability, the greater the number of fire missions assigned to *Crusader*s. This probability is effected by the intensity of battle.

r_c This parameter represents the probability that a target arrival is of type c. A type c target requires c *Crusader* pairs to execute the fire mission. The arrival of a type c target is effected by the phase of the battle and the force composition of the enemy. The number of *Crusader* elements required to attack the target is dependent on the *Crusader* rate of fire and lethality, and the size, disposition, and importance of the target. The Attack Guidance Matrix described in Chapter II of this thesis prescribes the pair to battalion size *Crusader* element to engage a type c target.

$\alpha(H(t); t)$ This function represents the probability that an alive *Crusader* pair is available to execute a fire mission. In battle, this parameter is a function of many factors (e.g., MTEFF, resupply time, movement time, etc.). In the MMC, the availability is a function of the probability of requiring repair or requiring ammunition resupply. A result of the MMCA, the long run proportion of time a *Crusader* pair is available to execute a fire mission, is used as the value of this parameter. A description of the MMCA is included in Chapter IV of this thesis.

$h_k(c)$ This parameter represents the probability a *Crusader* pair engaged in a fire mission of target type c is killed by counterfire. Fire missions requiring more than one *Crusader* pair may increase the probability that a single pair is killed by counterfire. In battle, this parameter is a function of many factors (scoot velocity, rate of fire, enemy sensor and counterfire capability, etc.).

B. MARKOV MODEL FOR *CRUSADER* AVAILABILITY PARAMETERS

Parameters

$p_m(t)$ — This parameter represents the stationary probability that an enemy target will arrive during a 5 minute time increment $(t, t + h]$. The greater the probability, the greater the number of fire missions assigned to *Crusader*s. This probability is effected by the intensity of battle.

y — This parameter represents the number of *Crusader* pairs alive. This parameter is exogenous to the MMCA model since there is no attrition.

p_b — This parameter represents the probability a *Crusader* pair needs ammunition resupply at the end of a 5 minute time increment. The larger the probability, the more time a *Crusader* pair spends being resupplied and, consequently, the less time a *Crusader* pair is available to execute fire missions.

p_{ba} — This parameter represents the probability a *Crusader* pair completes ammunition resupply at the end of a 5 minute time increment. The larger the probability, the less time a *Crusader* pair spends being resupplied and, consequently, the more time a *Crusader* pair is available to execute fire missions.

p_r — This parameter represents the probability a *Crusader* pair needs repair due to an essential function failure at the end of a 5 minute time increment. The larger the probability, the less time a *Crusader* pair spends being repaired and, consequently, the more time a *Crusader* pair is available to execute fire missions.

p_{ra} — This parameter represents the probability a *Crusader* pair completes repair at the end of a 5 minute time increment. The larger the probability, the less time a *Crusader* pair is under repair and, consequently, the more time a *Crusader* pair is available to execute fire missions.

r_c — This parameter represents the probability that a target arrival is of type c. A type c target requires c *Crusader* pairs to execute the fire mission. A detailed description of this parameter is included in Section A of this appendix.

C. INPUT PARAMETERS

Parameter

n_f This parameter represents the average number of rounds (projectiles) fired by a *Crusader* on a fire mission. The number of rounds fired is determined by the desired effects on the target and the lethality of a particular type round. More rounds are required to destroy a target then to suppress it. Standard data tables provide information on the effects of particular munitions on particular target types and the number required to attain a desired effect with a given probability. The ammunition available may limit the number of rounds fired. Ultimately, the guidance on the number of rounds to fire is published in the Attack Guidance Matrix before an operation.

θ This parameter represents the maximum rate at which *Crusader*s fire rounds in units of rounds per minute. The greater the rate of fire, the more rounds that can be fired in a specified time. This rate can be sustained for 5 minutes at which point it decreases because of the temperature of the *Crusader* gun. The rate is an engineering parameter constrained by the manufacturing characteristics of the gun.

N_s This parameter represents the number of independent fire missions a *Crusader* executes before it conducts a survivability move out of the enemy artillery counterfire footprint. The more missions executed before movement the more likely a *Crusader* is subject to detection and attack. The artillery commander determines this parameter based on weighing the need to provide continuous fires (uninterrupted by movement), the enemy detection capabilities and counterfire threat.

L_R This parameter represents the maximum number of rounds carried by a *Crusader*. It is a physical manufacturing characteristic of the *Crusader*.

α_R This parameter represents the percentage of the maximum number of rounds carried by a *Crusader* (L_R) at which time resupply is executed by a FARV. The percentage is determined by the artillery commander and is effected by the battle plan and the amount of available ammunition.

ρ This parameter represents the rate at which the maximum number of rounds carried by a *Crusader* (L_R) can be resupplied by a FARV

in units of rounds per minute. The rate is determined by the manufacturing characteristics of the FARV.

m_f — This parameter represents the mean-time-between-essential-function-failures (MTBEFF) in units of hours. An essential function failure prevents a *Crusader* from performing its mission (e.g., cannot deliver fires, cannot move, etc.). The MTBEFF is effected by strictly manufacturing characteristics.

m_r — This parameter represents the mean-time-to-repair (MTTR) a *Crusader* plus the administrative-and-logistics-downtime (ALDT) in units of hours. The MTTR is the time required to repair an essential function failure. The ALDT is the time a *Crusader* spends waiting for repair personnel or required repair parts.

C_{fp} — This parameter represents the counterfire footprint; the distance (in meters) a *Crusader* must move (in any direction) after firing to decrease to an appropriate level the probability of attrition by enemy counterfire. The distance is based on the lethality and accuracy capabilities of enemy artillery and sensors. The smaller the distance, the less likely a *Crusader* is attrited by enemy counterfire.

v_s — This parameter represents the scoot velocity; the rate at which a *Crusader* exits the counterfire footprint after conducting a fire mission in units of meters per second. The greater the scoot velocity, the less likely a *Crusader* is subject to attrition. The scoot velocity is dependent on the size of the counterfire footprint and the time required to move out of the footprint. The scoot velocity is computed as follows:

$$C_{fp} \,/\, \textit{time to exit footprint} \qquad \text{(D.1)}$$

The scoot velocity is affected, explicitly, by the speed and displacement time of the *Crusader*.

p_{acq} — This parameter represents the probability that, as a result of firing, a *Crusader* is detected by enemy sensors. This probability is effected by the firing signature and rate of fire of the *Crusader*, and the quantity and quality of enemy sensors.

$T_E(R)$ This parameter represents the median time for enemy artillery to engage a *Crusader* after acquiring the *Crusader*. This time begins to accumulate after enemy sensors detect a firing *Crusader* element. The smaller the time, the more likely enemy counterfire will result in *Crusader* attrition. The time is dependent on the quality of the enemy fire support system.

APPENDIX C. THE TRANSITION PROBABILITY MATRIX FOR THE MARKOV MODEL FOR *CRUSADER*

This appendix provides a three-step procedure to create the transition probability matrix, $P(t)$, for the Markov Model for *Crusader*. A numerical example is included to demonstrate the procedure.

A. THE MATRIX MKILL

Compute the square matrix:

$$M_K(c;j) = P\{j \text{ Crusader pairs are killed} | \text{mission is of type } c\} = \binom{c}{j} h_k(c)^j \left[1 - h_k(c)\right]^{(c-j)}$$

$$j = 0,1,\ldots,c \text{ and } M_K(c;j) = 0 \text{ for } j > c. \qquad \text{(C.1)}$$

The number of rows and columns are the number of different mission types. Each entry of the matrix is the probability j *Crusader* pairs are killed of the c that are assigned to a mission (binomial distribution). The probability of success, $h_k(c)$, is described in Chapter V.

$$M_K(c;j) = \begin{bmatrix} M_K(1;1) & M_K(1;2) & \ldots & M_K(1;C) \\ M_K(2;1) & M_K(2;2) & \ldots & M_K(2;C) \\ \vdots & \vdots & \ldots & \vdots \\ M_K(C;1) & M_K(C;2) & \ldots & M_K(C;C) \end{bmatrix} \qquad \text{(C.2)}$$

B. THE MATRIX KKILL

Compute the matrix $K_{ij}(t)$ where

$$K_{ij}(t) = P\{j \text{ Crusader pairs are killed} | i \text{ Crusader pairs are alive}\}.$$

If $j > i$

$$K_{ij}(t) = 0 \qquad \text{(C.3)}$$

and if $1 \leq j < i$

$$K_{ij}(t) = \sum_{c=j}^{\min(C,i)} r_c \sum_{k=c}^{i} \binom{i}{k} \alpha(i;t)^k \left(1 - \alpha(i;t)\right)^{i-k} M_K(c;j). \qquad \text{(C.4)}$$

The number of rows of the matrix is the number of *Crusader* pairs alive at $t = 0$. The number of columns of the matrix is the number of mission types, c.

Additionally, compute:

$$B_0(i;t) = P\{\text{zero crusader pairs are killed}|i \text{ are available and a mission arrives}\}$$

$$= 1 - \sum_{j=1}^{\min(i,C)} K_{ij}(t). \tag{C.5}$$

C. THE TRANSITION PROBABILITY MATRIX

Compute the entries for the transition probability matrix, $P(t)$, as follows:

$$Q_{ij}(t) = p_m(t)K_{i,i-j}(t) \tag{C.6}$$

$$Q_{ii}(t) = (1 - p_m(t)) + p_m(t)B_0(i;t) \tag{C.7}$$

$$q_i(t) = 1 - \sum_{j=1}^{H(0)} Q_{ij}(t). \tag{C.8}$$

$$P(t) = \left[\begin{array}{c|cccc} 1 & 0 & 0 & \cdots & 0 \\ \hline q_1(t) & Q_{11}(t) & Q_{12}(t) & \cdots & Q_{1H(0)}(t) \\ q_2(t) & Q_{21}(t) & Q_{22}(t) & \cdots & Q_{2H(0)}(t) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ q_{H(0)}(t) & Q_{H(0)1}(t) & Q_{H(0)2}(t) & \cdots & Q_{H(0)H(0)}(t) \end{array}\right] \tag{C.9}$$

D. NUMERICAL EXAMPLE

1. Input

$p_m(t)$	=	.3 ;
r_1	=	.5 ;
r_2	=	.5 ;
$\alpha(H(t) = 1;\, t)$	=	.928995 computed in Appendix E;
$\alpha(H(t) = 2;\, t)$	=	.897144 computed in Appendix E;
$\alpha(H(t) = 3;\, t)$	=	.928995 computed in Appendix E;
$h_k(c)$	=	.11262 computed in Chapter V;
$H(0)$	=	3 *Crusader* pairs at time $t = 0$.

A detailed definition of the input parameters is included in Appendix B.

2. Computations for the Matrix MKill

$$M_K(1;1) = \binom{1}{1}.11262^1[1-.11262]^{(1-1)} = .11262$$

$$M_K(2;1) = \binom{2}{1}.11262^1[1-.11262]^{(2-1)} = .19987$$

$M_K(1;2) = 0$

$M_K(2;2) = \binom{2}{2}.11262^2[1-.11262]^{(2-2)} = .01268$

$$M_K(c;j) = \begin{bmatrix} .11262 & 0 \\ .19987 & .01268 \end{bmatrix}$$

3. Computations for the Matrix Kkill

$K_{11}(t) = .5\left[1-\binom{1}{0}.928995^0(1-.9289951)^{1-0}\right]M_K(1,1) = .052312$

$K_{12}(t) = 0$

$$K_{21}(t) = .5\left[1-\binom{2}{0}.897144^0(1-.897144)^{2-0}\right]M_K(1,1)$$
$$+.5\left[1-\left(\binom{2}{0}.897144^0(1-.897144)^{2-0}+\binom{2}{1}.897144^1(1-.897144)^{2-1}\right)\right]$$
$$*M_K(2,1) = .580434$$

$$K_{22}(t) = .5\left[1-\binom{2}{0}.897144^0(1-.8971441)^{2-0}\right]M_K(1,2)$$
$$+.5\left[1-\left(\binom{2}{0}.897144^0(1-.897144)^{2-0}+\binom{2}{1}.897144^1(1-.897144)^{2-1}\right)\right]$$
$$*M_K(2,2) = .005103$$

$$K_{31}(t) = .5\left[1-\binom{3}{0}.928995^0(1-.928995])^{3-0}\right]M_K(1,1)$$
$$+.5\left[\left(\binom{3}{2}.928995^2(1-.928995)^{3-2}+\binom{3}{3}.928995^3(1-.928995)^{3-3}\right)\right]$$
$$*M_K(2,1) = .598495$$

$$K_{32}(t) = .5\left[1-\binom{3}{0}.928995^0(1-.928995)^{3-0}\right]M_K(1,2)$$
$$+.5\left[\binom{3}{2}.928995^2(1-.928995)^{3-2}+\binom{3}{3}.928995^3(1-.928995)^{3-3}\right]$$
$$*M_K(2,2) = .006249$$

$$K_{ij}(t) = \begin{bmatrix} .052312 & 0 \\ .580434 & .005103 \\ .598495 & .006249 \end{bmatrix}$$

4. Computations for the Transition Probability Matrix

$Q_{00}(t) = 1$

$Q_{11}(t) = (1-.3)+.3[1-K_{11}(t)] = .98431$

$Q_{22}(t) = (1-.3)+.3[1-(K_{21}(t)+K_{22}(t))] = .81858$

$$Q_{33}(t) = (1-.3)+.3\left[1-\left(K_{31}(t)+K_{32}(t)\right)\right] = .81858$$

$$Q_{21}(t) = .3K_{21} = .17413$$

$$Q_{23}(t) = 0$$

$$Q_{31}(t) = .3K_{32} = .00187$$

$$Q_{32}(t) = .3K_{31} = .17955$$

$$Q_{10}(t) = 1-\left(Q_{11}(t)+Q_{12}(t)+Q_{13}(t)\right) = .01569$$

$$Q_{20}(t) = 1-\left(Q_{21}(t)+Q_{22}(t)+Q_{23}(t)\right) = .00729$$

$$Q_{30}(t) = 1-\left(Q_{31}(t)+Q_{32}(t)+Q_{33}(t)\right) = 0$$

$$P(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ .01569 & .98431 & 0 & 0 \\ .00729 & .17413 & .81858 & 0 \\ 0 & .00187 & .17955 & .81858 \end{bmatrix}$$

APPENDIX D. COMPUTATIONAL FORMULAS FOR THE MARKOV MODEL FOR *CRUSADER*

This appendix presents the computational formulas of the MMC described in Chapter III. The computational formulas in this section were taken from Reference 8. These formulas provide for parametric sensitivity analysis of *Crusader* effectiveness. A numerical example is included to demonstrate the computations.

A. COMPUTATIONAL FORMULAS

1. The Expected Time Until *j* *Crusader* Pairs are Killed

Let

N_a = the number of time intervals during which there are *a* *Crusader* pairs alive.

Then if $\{ H(t); t = 0, h, 2h, \ldots \}$ is a Markov chain having stationary transition probabilities,

$$E\left[N_a | H(0) = i\right] = (I - Q)^{-1}_{i,a} \tag{D.1}$$

where I is an identity matrix with the same dimensions as Q and Q_{ij} is defined in (3.1). Notice that Q_{ij} is the submatrix of the transition probability matrix for the MMC.

Let the time until at least *j* *Crusader* pairs are killed be

$$T_K(j) = \min\left\{t \geq 0 : H(t) \leq H(0) - j\right\}.$$

Then the expected time there are $(j + 1)$ or more *Crusaders* alive given i, the initial number of *Crusader* pairs, is:

$$\begin{aligned} E\left[T_K(j) | H(0) = i\right] &= E\left[N_i + N_{i-1} + \ldots + N_{j+1} | H(0) = i\right] \\ &= \sum_{k=j+1}^{i} (I - Q)^{-1}_{i,k} \end{aligned} \tag{D.2}$$

2. The Expected Number of Missions Executed while *Crusaders* are Alive for a Markov Chain Model with Stationary Transition Probabilities

Let

$S(t)$ = the number of fire missions executed during $(0, t]$
$R(t)$ = the number of fire missions lost (rejected) during $(0, t]$ because not enough *Crusaders* were immediately available to fire the mission.

Then

$$m_s(a;t+h) = E[S(t+h)|H(0)=a] = \underbrace{\underbrace{p_m}_{\substack{\text{prob}\\ \text{mission}\\ \text{arrives}\\ \text{in } (0,h]}} \sum_{c=1}^{C} \underbrace{r_c}_{\substack{\text{prob}\\ \text{mission}\\ \text{of type c}}} \underbrace{\sum_{i=c}^{a} \binom{a}{i} \alpha(a)^i (1-\alpha(a))^{a-i}}_{\substack{\text{prob at least c Crusader}\\ \text{pairs are available}}}}_{\substack{\text{expected number of missions}\\ \text{executed during } (0,\, h]}}$$

$$+\sum_{j=1}^{a} \underbrace{Q_{aj}}_{\substack{\text{prob j}\\ \text{pairs are}\\ \text{alive after}\\ \text{the mission}}} \underbrace{m_s(j;t)}_{\substack{\text{expected number}\\ \text{of missions}\\ \text{executed during}\\ \text{the remaining}\\ \text{time intervals}\\ \text{given j pairs are}\\ \text{alive}}} \tag{D.3}$$

where if $c > a$

$$\sum_{i=c}^{a} \binom{a}{i} \alpha(a)^i (1-\alpha(a))^{a-i} = 0\,. \tag{D.4}$$

Let

$$m_s(a;\infty) = \lim_{t\to\infty} E[S(t)|H(0)=a]$$

$$= p_m \sum_{c=1}^{C} r_c \sum_{i=c}^{a} \binom{c}{i} \alpha(a)^i (1-\alpha(a))^{a-i} + \sum_{j=1}^{a} Q_{aj} m_s(j;\infty)$$

$$\equiv f_s(a) + \sum_{j=1}^{a} Q_{a,j} m_s(j;\infty). \tag{D.5}$$

Let $m_s(\infty)$ be a column vector with j^{th} entry $m_s(j;\infty)$. Then $m_s(a;\infty)$ can be rewritten as

$$\sum_{j} (I-Q)_{a,j} m_s(j;\infty) = f_s(a)\,. \tag{D.6}$$

Let I be an ($H(0)$ x $H(0)$) identity matrix. Then (I-Q)$^{-1}$ is a square matrix with $H(0)$ rows. Let f_s be a column vector with a entry $f_s(a)$. Then (I-Q)$^{-1}f_s$ is a column vector with ith entry

$$\sum_j (I-Q)_{ij}^{-1} f_s(j). \tag{D.7}$$

Therefore,

$$m_s(a,\infty) = \left[[I-Q]^{-1} f_s\right](a); \tag{D.8}$$

the a^{th} entry of the column vector $(I\text{-}Q)^{-1}f_s$ is the expected total number of fire missions executed given $H(0) = a$.

3. The Expected Number of Missions Lost while *Crusaders* are Alive for a Markov Chain Model with Stationary Transition Probabilities

Let

R(a) = the number of missions rejected while there are a *Crusader* pairs alive;

N(a) = the number of time intervals there are a *Crusader* pairs alive.

Then

$$E[R(a)|N(a)] = N(a)\left[\underbrace{p_m}_{\substack{\text{prob}\\\text{mission}\\\text{arrives}\\\text{in }(0,h]}} \sum_{c=1}^{C} \underbrace{r_c}_{\substack{\text{prob}\\\text{mission}\\\text{of type c}}} \underbrace{\sum_{i=0}^{c-1}\binom{a}{i}\alpha(a)^i(1-\alpha(a))^{a-i}}_{\substack{\text{prob fewer than c Crusader}\\\text{pairs are available}}}\right]. \tag{D.9}$$

Thus,

$$E[R(a)] = E\big[E[R(a)|N(a)]\big] = E[N(a)]p_m \sum_{c=1}^{C} r_c \sum_{i=0}^{c-1}\binom{a}{i}\alpha(a)^i(1-\alpha(a))^{a-i}. \tag{D.10}$$

Therefore, the total expected number of missions lost while there are *Crusaders* alive is:

$$E\left[\sum_{a=1}^{H(0)} R(a)|H(0)\right] = \sum_{a=1}^{H(0)} [I-Q]_{H(0),a}^{-1} p_m \sum_{c=1}^{C} r_c \sum_{i=0}^{c-1}\binom{a}{i}\alpha(a)^i(1-\alpha(a))^{a-i}. \tag{D.11}$$

4. Expected Number and Variance of *Crusader* Pairs Alive at Time *t*

The n-step transition probability matrix for (3.1) of the MMC is:

$$\tilde{\tilde{P}}_{ij}(0,t) = P\{H(t) = j | H(0) = i\} = \left[P(1) \times P(2) \times \ldots \times P(t)\right]_{ij}. \qquad \text{(D.12)}$$

Then, the expected number of *Crusader* pairs alive at time t is:

$$E\left[H(t) | H(0) = i\right] = \sum_j j\, \tilde{\tilde{P}}_{ij}(0,t). \qquad \text{(D.13)}$$

In order to determine the variance, first compute:

$$E\left[H(t)^2 | H(0) = i\right] = \sum_j j^2\, \tilde{\tilde{P}}(0,t) \qquad \text{(D.14)}$$

Then the variance in the expected number of *Crusader* pairs alive at time t, by definition, is:

$$Var\left[H(t) | H(0) = i\right] = E\left[H(t)^2 | H(0) = i\right] - E\left[H(t) | H(0) = i\right]^2. \qquad \text{(D.15)}$$

5. The Expected Total Number of Fire Missions Executed and Lost During (0, *t*] for Markov Chain Model with Non-Stationary Transition Matrices

First, the probability a fire mission is executed or lost during $(t, t+h]$ must be computed as follows:

$$p_{exec}(i;t) = \sum_j \underbrace{P\{H(t) = j | H(0) = i\}}_{\text{prob j Crusader pairs are alive at time t}} \underbrace{p_m}_{\text{prob mission arrivers}} \sum_{c=1}^{C} \underbrace{r_c}_{\text{prob mission of type c}} \underbrace{\sum_{k=c}^{j} \binom{j}{k} \alpha(j;t)^k \left[1-\alpha(j;t)\right]^{j-k}}_{\text{prob at least c Crusader pairs are available}}$$

$$\equiv \sum_j P\{H(t) = j | H(0) = i\} \beta_{exec}(j;t) \qquad \text{(D.16)}$$

and

$$p_{lost}(i;t) = \sum_j \underbrace{P\{H(t)=j|H(0)=i\}}_{\text{prob j Crusader pairs are alive at time t}} \underbrace{p_m}_{\text{prob mission arrives}} \sum_{c=1}^{C} \underbrace{r_c}_{\text{prob mission of type c}} \underbrace{\sum_{k=0}^{c-1}\binom{j}{k}\alpha(j;t)^k\left[1-\alpha(j;t)\right]^{j-k}}_{\text{prob fewer than c Crusader pairs are available}}$$

$$\equiv \sum_j P\{H(t)=j|H(0)=i\}\beta_{lost}(j;t). \tag{D.17}$$

Then the expected total number of missions executed during (0, t] given $H(0) = i$ is

$$m_{exec}(i;t) = \sum_{s=0}^{t-h} p_{exec}(i;s) \tag{D.18}$$

and the expected total number of missions lost during (0, t] given $H(0) = i$ is

$$m_{lost}(i;t) = \sum_{s=0}^{t-h} p_{lost}(i;s)\ . \tag{D.19}$$

B. NUMERICAL EXAMPLE

1. Input

$p_m(t)$	=	.3 ;
r_1	=	.5 ;
r_2	=	.5 ;
$\alpha(H(t) = 1; t)$	=	.928995 computed in Appendix E;
$\alpha(H(t) = 2; t)$	=	.897144 computed in Appendix E;
$\alpha(H(t) = 3; t)$	=	.928995 computed in Appendix E;
$h_k(\text{c})$	=	.11262;
$H(0)$	=	3 *Crusader* pairs at time $t = 0$.

2. Expected Time Until *J Crusaders* are Killed Given Start with Three *Crusader* Pairs

$$(I-Q)^{-1} = \begin{bmatrix} 63.734 & 0 & 0 \\ 61.1738 & 5.51207 & 0 \\ 61.2002 & 5.455 & 5.512 \end{bmatrix}$$

$$E\left[T_K(1)|H(0)=3\right] = 5.512$$

$$E\left[T_K(2)|H(0)=3\right] = 5.512 + 5.455 = 11.0575$$

$$E\left[T_K(3)|H(0)=3\right] = 61.2002 + 11.0575 = 72.2577$$

The expected time until all three *Crusader* pairs are killed is: 72.2577 five minute time increments (approximately 6 hours).

3. Expected Number of Missions Executed

$$f_s(1) = .3\left[.5\left(1-\binom{1}{0}.928995^0(1-.928995)^{1-0}\right)\right] = .139349$$

$$f_s(2) = .3\left[.5\left(1-\binom{2}{0}.897144^0(1-.897144)^{2-0}\right)+.5\binom{2}{2}.897144^2(1-.897144)^{2-2}\right] = .269143$$

$$f_s(3) = .3\,*$$

$$\left[.5\left(1-\binom{3}{0}.928995^0(1-.928995)^{3-0}\right)+.5\left(\binom{3}{2}.928995^2(1-.928995)^{3-2}+\binom{3}{3}.928995^3(1-.928995^{3-3}\right)\right]$$

$$= .297785$$

$$(I-Q)^{-1}f_s(j) = (I-Q)^{-1}\begin{bmatrix}.139349\\.269143\\.297785\end{bmatrix} = \begin{bmatrix}8.881405\\10.00806\\11.63786\end{bmatrix}$$

The expected number of missions executed = 11.63786.

4. Expected Number of Missions Lost

$$E\left[R(1)\middle|H(0)=3\right] = \left[I-Q\right]_{31}^{-1}.3\left[.5\binom{1}{0}.928995^0(1-.928995)^{1-0}+.5(1)\right] = 9.831858\ .$$

$$E\left[\mathrm{R}(2)\middle|H(0)=3\right] = \left[I-Q\right]_{32}^{-1}$$

$$*.3\left[.5\binom{2}{0}.897144^0(1-.897144)^{2-0}+.5(\binom{2}{0}.897144^0(1-.897144)^{2-0}+\binom{2}{1}.897144^1(1-.897144)^{2-1})\right]$$

$$=.168332$$

$$E\left[R(3)\middle|H(0)=3\right] = \left[I-Q\right]_{33}^{-1}$$

$$*.3\left[.5\binom{3}{0}.928995^0(1-.928995)^{3-0}+.5(\binom{3}{0}.928995^0(1-.928995)^{3-0}+\binom{3}{1}928995^1(1-.928995)^{3-1})\right]$$

$$=.01221$$

$$E\left[\sum_{a=1}^{3}R(a)\middle|H(0)=3\right] = 9.831858+.168332+.01221 = 10.012$$

The expected number of missions lost is 10.012.

5. Expected Number of *Crusaders* Alive After 20 Minutes of Battle (4 Time Intervals of 5 Minutes in Length)

$$P_{ij}(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ .01569 & .98431 & 0 & 0 \\ .00729 & .17413 & .81858 & 0 \\ 0 & .00187 & .17955 & .81858 \end{bmatrix}$$

$$P_{ij}^2(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ .031134 & .968866 & 0 & 0 \\ .01599 & .313937 & .670073 & 0 \\ .001338 & .034636 & .293952 & .670073 \end{bmatrix}$$

$$P_{ij}^3(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ .046335 & .953665 & 0 & 0 \\ .0258 & .425691 & .548509 & 0 \\ .004025 & .086532 & .360935 & .548509 \end{bmatrix}$$

$$P_{ij}^4(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ .061298 & .938702 & 0 & 0 \\ .036748 & .514524 & .448998 & 0 \\ .008014 & .14905 & .393939 & .448998 \end{bmatrix}$$

$$E\left[H(20\min)\middle|H(0)=3\right] = 0(.008014) + 1(.14905) + 2(.393939) + 3(.448998) = 2.28392$$

The expected number of *Crusader* pairs alive after 20 minutes is: 2.28392.

$$Var\left[H(20\min)\middle|H(0)=3\right] = \left[0^2(.008014) + 1^2(.14905) + 2^2(.393939) + 3^2(.448998)\right]$$
$$-2.28392^2 = .54950$$

The standard error in the expected number of *Crusader* pairs alive after 20 minutes of battle is: $\sqrt{.54950} = .74128$ *Crusader* pairs.

6. The Expected Total Number of Missions Lost in 20 Minutes

For this section and the proceeding section in this appendix the cumulative binomial distribution computational form will be represented by the following notation: bin(number of successes, number of trials, probability of success). Then the expected total number of fire mission lost during 20 minutes of battle is:

$$p_L(0) = (.3)[.5\text{bin}(0,3,.928995)+.5\text{bin}(1,3,.928995)] = .002215$$

$$\begin{aligned} p_L(5\text{ min}) &= .0(.3)[.5(1)+.5(1)]+.00187(.3)[(.5)\text{bin}(0,1,.928995)+.5(1)] \\ &+.17955(.3)[.5\text{bin}(0,2,.897144)+.5\text{bin}(1,2,.897144] \\ &+.81858(.3)[.5\text{bin}(0,3,.928995)+.5\text{bin}(1,3,.928995)] = .00765 \end{aligned}$$

$$\begin{aligned} p_L(10\text{ min}) &= .001338(.3)[.5(1)+.5(1)]+.034636(.3)[(.5)\text{bin}(0,1,.928995)+.5(1)] \\ &+.293952(.3)[.5\text{bin}(0,2,.897144)+.5\text{bin}(1,2,.897144] \\ &+.670073(.3)[.5\text{bin}(0,3,.928995)+.5\text{bin}(1,3,.928995)] = .02013 \end{aligned}$$

$$\begin{aligned} p_L(15\text{ min}) &= .004025(.3)[.5(1)+.5(1)]+.086532(.3)[(.5)\text{bin}(0,1,.928995)+.5(1)] \\ &+.360935(.3)[.5\text{bin}(0,2,.897144)+.5\text{bin}(1,2,.897144] \\ &+.548509(.3)[.5\text{bin}(0,3,.928995)+.5\text{bin}(1,3,.928995)] = .02746 \end{aligned}$$

$m_L(20\text{ min}) = p_L(0) + p_L(5) + p_L(10) + p_L(15) = .07739.$
The expected total number of missions lost in 20 minutes is .07739.

7. The Expected Total Number of Missions Executed in 20 Minutes

$$p_S(0) = (.3)[.5(1-\text{bin}(0,3,.928995))+.5(1-\text{bin}(1,3,.928995))] = .297785$$

$$\begin{aligned} p_S(5) &= 0(.3)[.5(0)+.5(0)]+.00187(.3)[(.5)(1-\text{bin}(0,1,.928995))+.5(0))] \\ &+.17955(.3)[.5(1-\text{bin}(0,2,.897144))+.5(1-\text{bin}(1,2,.897144))] \\ &+.81858(.3)[.5(1-\text{bin}(0,3,.928995))+.5(1-\text{bin}(1,3,.928995))] = .29235 \end{aligned}$$

$$\begin{aligned} p_S(10) &= .001338(.3)[.5(0)+.5(0)]+.034636(.3)[(.5)(1-\text{bin}(0,1,.928995))+.5(0))] \\ &+.293952(.3)[.5(1-\text{bin}(0,2,.897144))+.5(1-\text{bin}(1,2,.897144))] \\ &+.670073(.3)[.5(1-\text{bin}(0,3,.928995))+.5(1-\text{bin}(1,3,.928995))] = .28348 \end{aligned}$$

$$\begin{aligned} p_S(15) &= .004025(.3)[.5(0)+.5(0)]+.086532(.3)[(.5)(1-\text{bin}(0,1,.928995))+.5(0))] \\ &+.360935(.3)[.5(1-\text{bin}(0,2,.897144))+.5(1-\text{bin}(1,2,.897144))] \\ &+.548509(.3)[.5(1-\text{bin}(0,3,.928995))+.5(1-\text{bin}(1,3,.928995))] = .27254 \end{aligned}$$

$p_s(20\text{ min}) = p_s(0) + p_s(5) + p_s(10) + p_s(15) = .84837.$

The expected number of missions executed is .84837.

APPENDIX E. COMPUTATIONS FOR THE MARKOV MODEL FOR *CRUSADER* AVAILABILITY

This appendix provides the computations for determining, $\alpha(H(t); t)$, the long-run availability of a *Crusader* pair. The value is used in the MMC. A numerical example of the computations is included, also.

A. COMPUTATIONS

The long-run proportion of time a *Crusader* pair is available given y *Crusader* pairs are alive is:

$$\alpha(y) = \lim_{n \to \infty} \overline{P}(y)^n_{AA} \tag{E.1}$$

where $\overline{P}(y)$ is defined in (4.1).

Let

$$\pi(j) = \lim_{n \to \infty} \overline{P}(y)^n_{ij} \tag{E.2}$$

be the limiting distribution for the Markov chain with transition matrix (4.1). Therefore, $\{\pi(j); j = A, B, R\}$ satisfies the matrix equations

$$\begin{aligned} \pi \overline{P}(y) &= \pi \\ \sum_j \pi(j) &= 1 \end{aligned} \qquad \text{(E.3),(E.4)}$$

where π is a row vector. To solve the equations numerically rewrite (E.3) as follows:

$$0 = \pi\left(I - \overline{P}\right) \tag{E.5}$$

where I is an identity matrix with the same dimensions as $\overline{P}$ and 0 is a row vector of

zeros. Replace the third column of $(I-\overline{P})$ with 1's. Solve with matrix operations for π as follows:

$$[0,0,1]=[\pi(A),\pi(B),\pi(R)]\begin{bmatrix} p_m\sum_{c=1}^{C} r_c f\left(\frac{c}{y}\right)[p_b+p_r] & -p_m\sum_{c=1}^{C} r_c f\left(\frac{c}{y}\right)p_b & 1 \\ -p_{ba} & p_{ba} & 1 \\ -p_{ra} & 0 & 1 \end{bmatrix}. \quad \text{(E.6)}$$

Solve for $\pi(A),\pi(B),\pi(R)$. The long run availability $\pi(y)=\pi(A)$.

It should be recognized that if the parameters in the model are time-dependent then there will not be a long-run availability. In this case, solve for π(y) by matrix multiplication of (4.1) and use the value in (3.1).

B. NUMERICAL EXAMPLE

A detailed definition of the parameters which follow is included in Appendix B.

1. Parameters

$p_m(t)$	=	.3 ;
r_1	=	.5 ;
r_2	=	.5 ;
y	=	1,2,3 *Crusader* pairs;
p_b	=	.1111 computed in Chapter V;
p_{ba}	=	.347222 computed in Chapter V;
p_r	=	.00245 computed in Chapter V;
p_{ra}	=	.01292 computed in Chapter V.

2. The Transition Probability Matrix

For the case where the number of *Crusader* pairs alive is 2:

$$\overline{P}(2)=\begin{bmatrix} 1-.3(.5(1/2)(.1111+.00245)+.5(2/2)(.1111+.00245)) & .3(.5(1/2).1111+.5(2/2).1111) & .3(.5(1/2).00245+.5(2/2).00245) \\ .347222 & .65278 & 0 \\ .01292 & 0 & .98708 \end{bmatrix}$$

$$I-\overline{P}(2)=\begin{bmatrix} .02555 & -.0250 & 1 \\ -.347222 & .65278 & 1 \\ -.01292 & 0 & 1 \end{bmatrix};$$

$$\begin{bmatrix}0 & 0 & 1\end{bmatrix}\left[I-\overline{P}(2)\right]^{-1}=\left[\pi(A_2) \quad \pi(B_2) \quad \pi(R_2)\right];$$

$$\begin{bmatrix}0 & 0 & 1\end{bmatrix}\left[I-\overline{P}(2)\right]^{-1}=\left[.897144 \quad .064588 \quad .038268\right].$$

$\overline{P}(1)$ and $\overline{P}(3)$ are computed similarly to provide the long-run probabilities for *Crusader* pairs available. The cumulative computations result in the following matrix:

$$\begin{array}{rr} & \begin{array}{ccc}\text{Avail} & \text{Resupply} & \text{Repair}\end{array} \\ \begin{array}{r} 1 \\ \text{Crusaders Alive } 2 \\ 3 \end{array} & \begin{bmatrix} .928995 & .044588 & .026418 \\ .897144 & .064588 & .038268 \\ .928995 & .044588 & .026418 \end{bmatrix} \end{array}$$

where $\alpha(H(t) = 2;\, t) = .897144$ in the MMC.

APPENDIX F. INSTRUCTIONAL DOCUMENTATION FOR THE CBSI

This appendix presents the instructions included in a "readme.doc" file for the CBSI.

1. Enter a battletime into the input dialog box in whole, integer hours from 1 to 96.

2. Enter all input in appropriate units as stated on the CBSI input page.

3. If an error occurs as a result of improper input, select "End" from the "pop up" dialog box then check to ensure all input is entered correctly.

4. The probabilities of a type *c* mission arriving must sum to 1.

5. The sheet with the name "GraphicsPage" is a blank sheet which may be used to construct graphs.

6. Enter all input data, as desired, then use the "RECOMPUTE" or "FINITE-TIME" button to invoke calculations.

7. Each sheet in the CBSI is named for organizational purposes. The name identifies the component of the CBSI which appears on the specified sheet. The sheet names are as follows:

    a. Input_Output: user input and output page
    b. GraphicsPage: page available for graphics construction
    c. IntComp: intermediate computations page
    d. FiniteTimeComp: finite-time computations page
    e. VBFiniteTime: *Visual Basic* program module

8. The "VBFiniteTime" sheet is protected to prevent the program from being changed, inadvertently. The password to change the protection status is: "cbsi" (no quotations). The procedure to "unprotect" the sheets is as follows:

    a. select the "VBFiniteTime" sheet;
    b. select the **Tools** menu;
    c. choose **Protection**;
    d. select **Unprotect Worksheet**.

APPENDIX G. CRUSADER BATTALION SPREADSHEET IMPLEMENTATION COMPUTATIONAL PAGE EXAMPLE

This appendix presents an example of an intermediate computational page from the CBSI for the "base case" *Crusader* data file. Figure 12 displays the results of intermediate calculations for the supporting models of the MMC. These results are used in the final calculations for the MMC.

Intermediate Computational Results for the CBSI

Time for Mission =		0.4	(nf/theta)		
Probability 1 Crusader Pair Engaged in Fire Mission of Target Type c is Killed:			**hk(c)**		
Pair	**c =1**	0.112639	0.112639	**Probability Red Acquires Blue =**	0.91
Platoon	**c =2**	0.199903			
	c =3	0.266079		**Shoot and Scoot Time =**	1.906325
Battery	**c =4**	0.314811			
	c =5	0.349189			
	c =6	0.371828		**Rate at Which Red Engages**	
	c =7	0.384936		**Blue =**	0.069315
Battery	**c =8**	0.390374			
	c =9	0.389703			
	c =10	0.384231			
	c =11	0.375046			
Battalion	**c =12**	0.363056			
	Probability Crusader Pair Needs Resupply =		0.111111		
	Probability Crusader Pair Completes Resupply =		0.347222		
	Probability Crusader Pair Needs Repair =		0.002451		
	Probability Crusader Pair Completes Repair =		0.01292		

Figure 12. Intermediate Computational Results from the CBSI

APPENDIX H. VISUAL BASIC FINITE-TIME COMPUTATIONS CODE

```
Sub FiniteTime()

Dim time As String
Dim counter As Integer
i = 4
j = 16
sumexecute = 0
sumlost = 0
time = InputBox("Provide number of hours of battle (min 1: max 96)")
steps = CInt(time) * 12
Sheets("Input_Output").Select
Cells(88, 4).Select
Selection.Value = time
Calculate

'Computes the P(2) matrix
 Sheets("FiniteTimeComp").Select
 Range("a4:m16").Select
 Selection.FormulaArray = "=mmult(transmatrix,transmatrix)"

'Computes the P(3) to P(n) matrix and cummulates expected missions
'lost and executed
 For counter = 1 To steps - 3
   i = i + 14
   j = j + 14
   Range(Cells(i, 1), Cells(j, 13)).Select
   Selection.FormulaArray = "=mmult(r[-14]c:r[-2]c[12],transmatrix)"

   Cells(i + 12, 14).Select
   Selection.FormulaArray = "=mmult(rc[-13]:rc[-1],probmsnlostmatrix)"
   sumlost = sumlost + Selection.Value

   Cells(i + 12, 15).Select
   Selection.FormulaArray = "=mmult(rc[-14]:rc[-2],probmsnexecutematrix)"
   sumexecute = sumexecute + Selection.Value

 Next counter

'Computes partial missions lost and executed in battle time
'(does not include lost/executed at t=0,5,10 minutes)
 Cells(39, 17).Select
 Selection.Value = sumlost
 Cells(41, 17).Select
```

```
 Selection.Value = sumexecute

'Computes expected value and variance of Crusaders alive
 Range(Cells(i + 12, 1), Cells(j, 13)).Select
 Names.Add Name:="temp", RefersToR1C1:=Selection
 Cells(24, 15).Select
 Selection.FormulaArray = "=mmult(temp,Numbermatrix)"
 Cells(19, 15).Select
 Selection.FormulaArray = "=mmult(temp,Squarematrix)"
 Cells(22, 15).Select
 Selection.Formula = "=o19-o24^2"

'Recomputes spreadsheet values
 Calculate
 Sheets("Input_Output").Select
 Cells(96, 6).Select

End Sub
```

APPENDIX I. *PALADIN* CBSI INPUT PAGE

Crusader Battalion Spreadsheet Implementation
*** 12 Crusader Pairs (24 Total Crusaders)**
*** Pair to Battalion Fire Missions**

User Provided Input :

Probability Fire Mission Arrives: 0.3

Probability Fire Mission is Target Type c Requiring c Crusader Pairs to Fire:

Pair Mission	0.01
Platoon Mission	0.75
	0.01
Battery Mission	0.1
	0.01
	0.01
	0.002
Battery Mission	0.002
	0.002
	0.002
	0.002
Battalion Mission	0.1

Scoot Speed(m/s)= (750m/90s=8.3m/s) 6.22

Counterbattery Footprint(meters)= 560

Crusader Payload = 39

Median Time until Red Engages Blue = (min) 10

Number of Rounds Fired per Mission = 4

Rate of Fire (rounds/min) = 4

Number of Missions Fired Before Survivability Move = 1

Resupply Ammunition at Percentage Expended (decimal percent) = 0.6

Ammunition Transfer Rate (rnds/min)= 1.714

Mean Time Between Essential Function Failures (hours) = 21

Mean Time to Repair plus ALDT = (hours) 6.45

Probability of Acquisition = of Crusaders by enemy Sensors as a Result of Firing 0.7

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